LAB 4

Inverse Kinematics

4.1 Important

Read the entire lab before starting and especially the "Grading" section so you are aware of all due dates and requirements associated with the lab.

4.2 Objectives

The purpose of this lab is to derive and implement a solution to the inverse kinematics problem for the UR3 robot. In this lab we will:

- Derive elbow-up inverse kinematic equations for the UR3
- Write a Python function that moves the UR3 to a point in space specified by the user.

4.3 Reference

Chapter 6 of *Modern Robotics* provides multiple examples of inverse kinematics solutions.

4.4 Tasks

4.4.1 Solution Derivation

Make sure to read through this entire lab before you start deriving your solution. There are some needed details not covered in this section.

Given a desired end-effector position in space $(x_{grip}, y_{grip}, z_{grip})$ and orientation $\{\theta_{yaw}, \theta_{pitch}(fixed), \theta_{roll}(fixed)\}$, write six mathematical expressions that yield values for each of the joint angles. For the UR3 robot, there are many solutions to the inverse kinematics problem. We will implement only one of the *elbow-up* solutions.

- In the inverse kinematics problems you have examined in class (for 6 DOF arms with spherical wrists), usually the first step is to solve for the coordinates of the wrist center. The UR3 does not technically have a spherical wrist center but we will define the wrist center as z_{cen} which equals the same desired z value of the suction cup and x_{cen}, y_{cen} are the coordinates of θ_6's z axis. In addition, to make the derivation manageable, add that θ_5 will always be $-90°$ and θ_4 is set such that link 7 and link 9 are always parallel to the world x,y plane.

- Solve the inverse kinematics problem in the following order:

  1. x_{cen}, y_{cen}, z_{cen}, given yaw desired in the world frame and the desired x,y,z of the suction cup. The suction cup aluminum plate (link 9) has a length of 0.0535 meters from the center line of the suction cup to the center line of joint 6. Remember that this aluminum plate should always be parallel to the world's x,y plane. See Figure 4.2.
  2. θ_1, by drawing a top down picture of the UR3, Figure 4.1, and using x_{cen}, y_{cen}, z_{cen} that you just calculated.
  3. θ_6, which is a function of θ_1 and yaw desired. Remember that when θ_6 is equal to zero the suction cup aluminum plate is parallel to link 4 and link 6.
  4. x_{3end}, y_{3end}, z_{3end} is a point off of the UR3 but lies along the link 6 axis, Figure 4.1. For example if $\theta_1 = 0°$ then $y_{3end} = 0$. If $\theta_1 = 90°$ then $x_{3end} = 0$. First use the top down view of the UR3 to find x_{3end}, y_{3end}. One way is to choose an appropriate coordinate frame at x_{cen}, y_{cen} and find the translation matrix that rotates and translates that coordinate frame to the base frame. Then find the vector in the coordinate frame you chose at x_{cen}, y_{cen} that points from x_{cen}, y_{cen} to x_{3end}, y_{3end}. Simply multiply this vector by your translation matrix to find the world coordinates at x_{3end}, y_{3end}. For z_{3end} create a view of the UR3, Figure 4.2, that is a projection of the robot onto a plane perpendicular to the x,y world frame and rotated by θ_1 about the base frame. Call this the side view. Looking at this side view you will see that z_{3end} is z_{cen} offset by a constant.
  5. θ_2, θ_3 and θ_4, by using the same side view drawing just drawn above to find z_{3end}, Figure 4.2. Now that x_{3end}, y_{3end}, z_{3end} have been found use sine, cosine and the cosine rule to solve for partial angles that make up θ_2, θ_3 and θ_4. Hint: In this side view, a parallel to the base construction line through joint 2 and a parallel to the base construction line through joint 4 are helpful in finding the needed partial angles.

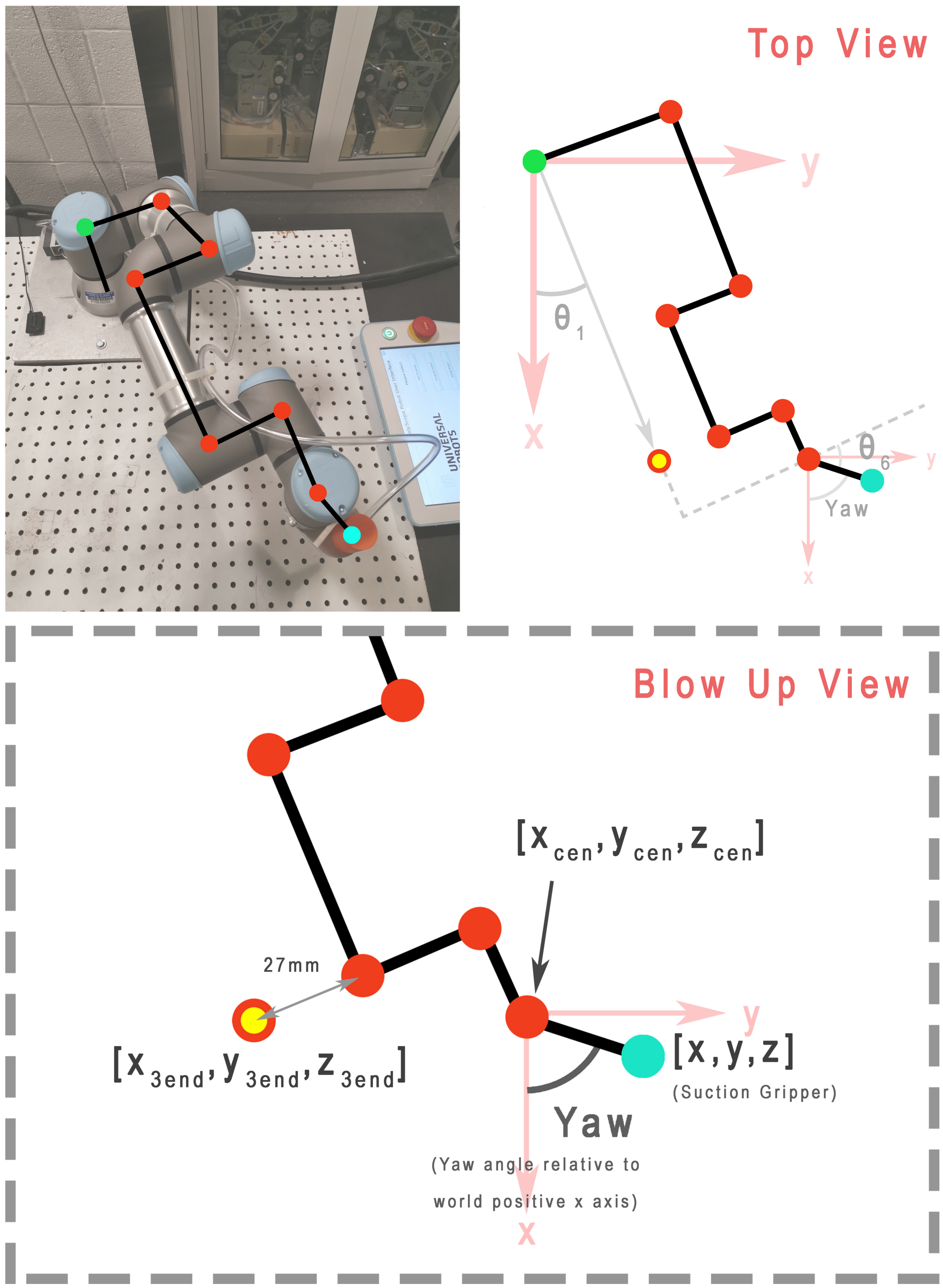

Figure 4.1: Top View Stick Pictorial of UR3. Note that the coordinate frames are in the same direction as the World Frame but not at the World frame's origin. One origin is along the center of joint 1 and the second is along the center of joint 6.

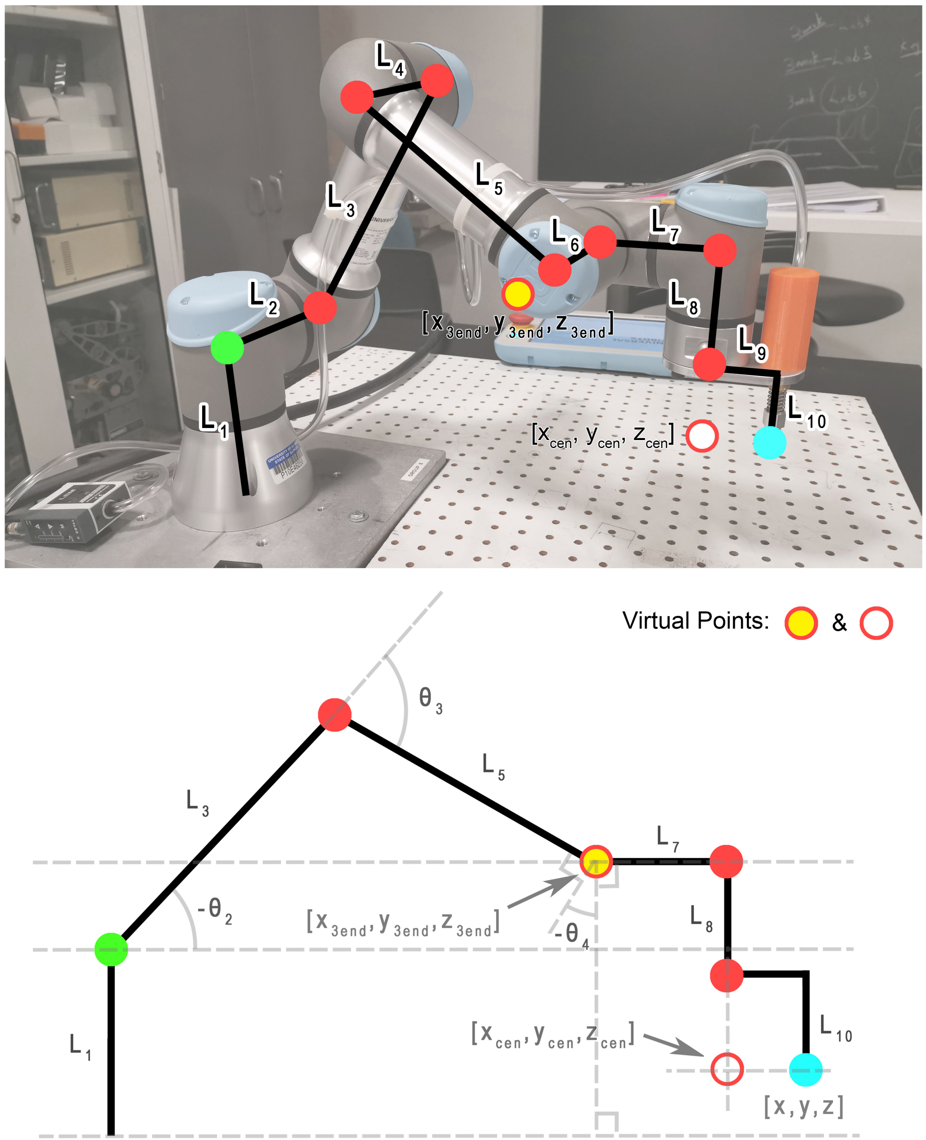

Figure 4.2: Side View Stick Pictorial of UR3.

4.4.2 Implementation

Implement the inverse kinematics solution by writing a Python function to receive world frame coordinates $(x_{Wgrip}, y_{Wgrip}, z_{Wgrip}, yaw_{Wgrip})$, compute the desired joint variables $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}$, and command the UR3 to move to that pose using functions written in Lab4.

4.5 Procedure

- Download **lab4Py.tar.gz** from the course website and extract it in your "**src**" directory. You will notice that there are three .py files. **lab4_exec.py**, **lab4_func.py** and **lab4_header.py**. The **lab4_func.py** file again will be compiled into a library so that future labs can easily call the inverse kinematic function. Like Lab 3, most of the needed code is given to you in **lab4_exec.py**. Your main job will be to add all the inverse kinematic equations to **lab4_func.py**. Please refer to the intermediate steps below to perform the inverse kinematic calculations. If you look at **lab4_exec.py** it includes **lab4_func.py**. This allows you to call the functions you created in **lab4_func.py**.

- Once your code is finished, run it using "**rosrun lab4pkg_py lab4_exec.py [x] [y] [z] [yaw(degrees)]**" - e.g. "**rosrun lab4pkg_py lab4_exec.py 0.1 0.1 0.15 90**". Remember that in another command prompt you should have first run roscore and drivers using "**roslaunch ur3_driver ur3_driver.launch**".

- You should measure the x,y,z position of the end-effector using the provided ruler and square.

- You should verify that your code works by selecting a variety of poses that will test the full range of motion. Your TA will not be providing you test points.

- In your code (This is repeating the derivation steps above):

  1. Establish the world coordinate frame (frame w) centered at the corner of the UR3's base shown in Figure 4.3. The x_w and y_w plane should correspond to the surface of the table, with the x_w axis parallel to the sides of the table and the y_w axis parallel to the front and back edges of the table. Axis z_w should be normal to the table surface, with up being the positive z_w direction and the surface of the table corresponding to $z_w = 0$.
     We will solve the inverse kinematics problem in the base frame (frame 0), so we will immediately convert the coordinates entered by the user to base frame coordinates. Write three equations relating coordinates $(x_{Wgrip}, y_{Wgrip}, z_{Wgrip})$ in the world frame to coordinates

$(x_{grip}, y_{grip}, z_{grip})$ in the base frame of the UR3.

$$x_{grip}(x_{W grip}, y_{W grip}, z_{W grip}) =$$
$$y_{grip}(x_{W grip}, y_{W grip}, z_{W grip}) =$$
$$z_{grip}(x_{W grip}, y_{W grip}, z_{W grip}) =$$

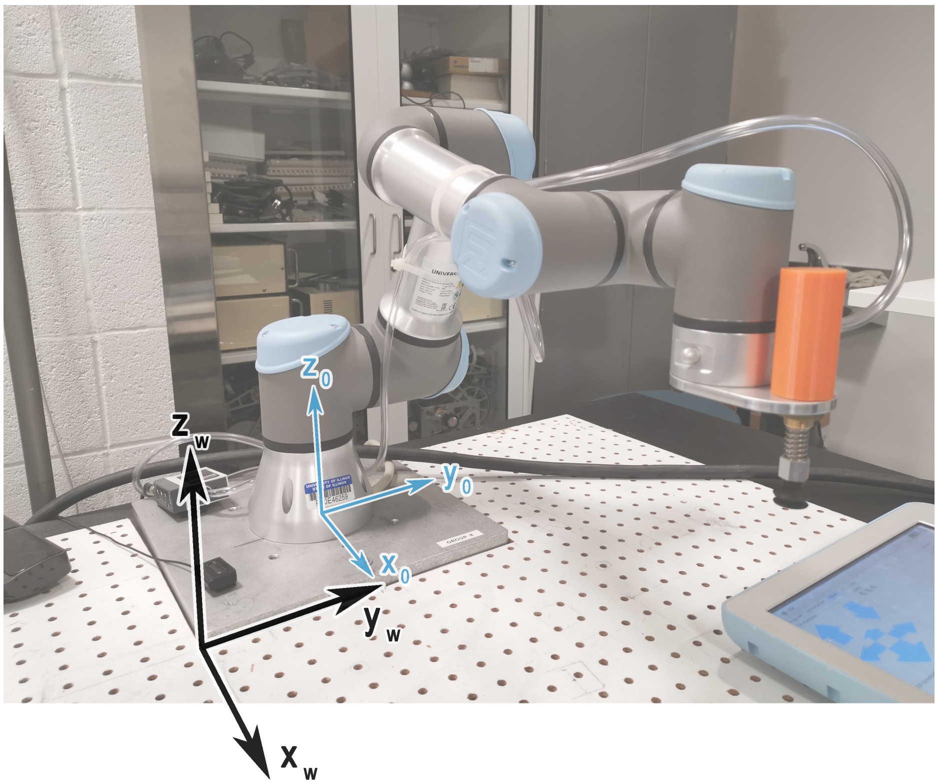

Figure 4.3: Correct location and orientation of the world frame.

2. Given the desired position of the gripper $(x_{grip}, y_{grip}, z_{grip})$ (in the base frame) and the yaw angle, find wrist's center point $(x_{cen}, y_{cen}, z_{cen})$.

$$x_{cen}(x_{grip}, y_{grip}, z_{grip}, yaw) =$$
$$y_{cen}(x_{grip}, y_{grip}, z_{grip}, yaw) =$$
$$z_{cen}(x_{grip}, y_{grip}, z_{grip}, yaw) =$$

3. Given the wrist's center point $(x_{cen}, y_{cen}, z_{cen})$, write an expression for the waist angle θ_1. Make sure to use the **atan2()** function instead of **atan()** because **atan2()** takes care of the four quadrants the x,y coordinates could be in.

$$\theta_1(x_{cen}, y_{cen}, z_{cen}) = \tag{4.1}$$

4. Solve for the value of θ_6, given yaw and θ_1.

$$\theta_6(\theta_1, yaw) = \tag{4.2}$$

5. Find the projected end point $(x_{3end}, y_{3end}, z_{3end})$ using $(x_{cen}, y_{cen}, z_{cen})$ and θ_1.

$$x_{3end}(x_{cen}, y_{cen}, z_{cen}, \theta_1) =$$
$$y_{3end}(x_{cen}, y_{cen}, z_{cen}, \theta_1) =$$
$$z_{3end}(x_{cen}, y_{cen}, z_{cen}, \theta_1) =$$

6. Write expressions for θ_2, θ_3 and θ_4 in terms of the end point. You probably will want to define some intermediate variables to help you with these calculations.

$$\theta_2(x_{3end}, y_{3end}, z_{3end}) =$$
$$\theta_3(x_{3end}, y_{3end}, z_{3end}) =$$
$$\theta_4(x_{3end}, y_{3end}, z_{3end}) =$$

7. Now that your code solves for all the joint variables (remember that θ_5 is always $-90°$) send these six values to the Lab 3 function **lab_fk()**. You will need to copy you Lab 3 solution into these functions. Do this simply to check that your inverse kinematic calculations are correct. Double check that the x,y,z point that you asked the robot to go to is the same value displayed by the forward kinematic equations.

4.6 Report

You should submit a lab report using the guidelines given in the ECE 470: How to Write a Lab Report document. Please be aware of the following:

- Lab reports will be submitted online at GradeScope.

Your lab report should include the following:

- A clearly written derivation of the inverse kinematics solution for each joint variable $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$. You **must** include figures in your derivation. Diagrams should be your own creation and clear and easily read. Do not use hand drawn figures or annotations.
- For each test point include:
  - The given $\{(x_{w_{grip}}, y_{w_{grip}}, z_{w_{grip}}), \theta_{yaw}\}$
  - The measured position
  - The scalar error
- Include a brief discussion of sources of error.

4.7 Demo

Your TA will require you to run your program twice, each time with a different set of desired position and orientation. Your program should reach the desired position and orientation with almost no error.

4.8 Grading

- 75 points, successful demonstration.
- 20 points, individual report.
- 5 points, attendance.