

# 1 Lab 3: DC Motor Transfer Function Estimation by Explicit Measurement

## 1.1 Introduction

There are three common methods for determining a plant's transfer function. They are:

1. Measure all the physical parameters of the system used to derive the equations. Then compute the overall transfer function. – (Lab 3)
2. Treat the system as a black box and use:
  - frequency response, or
  - step responsemethods to determine DC gain, poles, and zeros of the system. – (Lab 4)

The following physical quantities of the DC Motor need to be measured to determine its transfer function:

- Armature resistance:  $R_a$  [ohms]
- Armature inductance:  $L_a$  [Henries]
- Torque constant:  $K_T$  [N-m/amp]. Proportionality constant that relates Torque and current. Please refer to equation 5.
- Back EMF constant:  $K_b$  [volts-s/rad]. Proportionality constant that relates Angular velocity and back e.m.f. . Please refer to equation 2.
- Viscous friction coefficient:  $B$  [N-m-s/rad]
- Rotor moment of inertia:  $J$  [kg-m<sup>2</sup>]

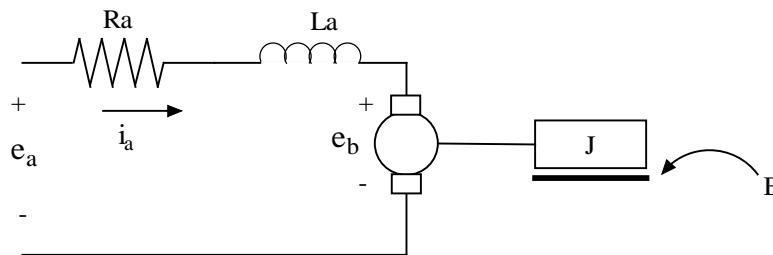


Figure 1: D.C. Motor Diagram

This lab focuses on measuring motor properties by manipulating the system in order to reduce its describing equations, permitting parameter measurement via the output characteristics from a given input. For example, we might stop the motor from rotating and thus nullify all velocity terms from the equations of motion.

## 1.2 Pre-lab

1. In your own words, define what a **mathematical model** is. Why are mathematical models important in engineering?
2. Describe (with figures) two methods for determining the time constant of an exponential decay.
3. Based on conservation of energy principals, set mechanical power equal to electrical power and show that  $K_b = K_T$ . (Use SI units for all your calculations). The easiest way to do this is to think of the situation where the motor+ and motor- leads are shorted together,  $e_a = 0$ . In this case if you manually move the motor the only voltage generated in the circuit is due to the back-emf of the motor. Recall that in this case,  $P_{el} = E_B * I_A$  and  $P_{mech} = T_m * \omega$ .
4. Write the 4 electrical and mechanical equations that may be used to find the transfer function of the DC motor  $\omega(s)/E_a(s)$ . Use Figure 1, the equations provided in this lab, and the example mentioned in section 1.4 of this lab as a guide.
5. Using the equations in question 4, describe how you would measure:
  - a. Viscous friction
  - b. Rotor moment of inertia
  - c. Armature resistance
6. Compute the voltage drop across  $R_s$  in the circuit shown in Figure 2. Also find the time constant for this R-L circuit

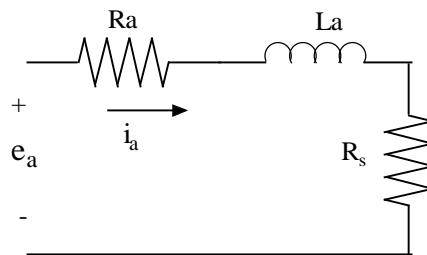


Figure 2: RL circuit

## 1.3 Objectives

By the end of this laboratory experiment, you will be able to:

- Understand the process of gray box identification
- Estimate a system transfer function from laboratory measurements

## 1.4 References

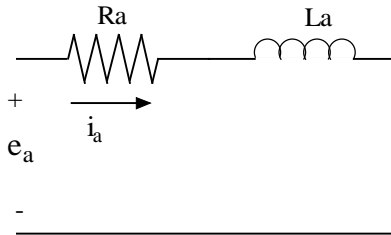
See Chapter 3 of *Feedback Control Systems* (2011), Fifth ed. by Charles L. Phillips and John M. Parr. More specifically Section 3.2 and Example 3.3 will be useful references for this lab.

Note: the same information is in *Feedback Control Systems* (2000), Fourth ed. by Charles L. Phillips and Royce D. Harbor.

## 1.5 Armature Resistance: $R_a$

**Public Service Announcement:** Many of the values obtained in this lab will need to be recorded and saved for future use. Please record all coefficient and parameter values on your data sheet and also on something that you will not turn in with a lab report and bring the values to subsequent labs.

Equation 1 (see below) describes the relationship between the input voltage, the armature resistance, armature inductance and the back EMF. If the rotor is restricted from rotating (see Figure 3), the back EMF will become zero (due to the fact that back EMF is proportional to angular velocity,  $\omega$ ). Similarly, the armature inductance term will reduce to zero once steady state conditions have been reached. Hence, by measuring the input voltage and the resultant current we can measure  $R_a$ . Follow the directions below:



**Figure 3: Model of locked motor**

- Connect the motor, flywheel and rotor locking device. Connect the HP 0-20V power supply to the motor.



- Turn on all the equipment (don't forget the HP 6632A programmable power supply) and then open the Agilent VEE file: "`n:/labs/ge320/exp3/ra.vxe`"

- From the computer, “Reset” the power supply and then switch “Output Enable” off. Set the current to 2.5 amps and the voltage to 5V. The amplifier is set to *constant voltage mode* and hence it will maintain the set voltage to the motor for any required amperage to a MAXIMUM of 2.5 amps.
- At the Agilent VEE console, toggle the “Output Enable” of the power supply to “ON”. This enables the power supply’s voltage. You can look at the power supply’s display and see what actual voltage and current is being applied. Record these values and those obtained by incrementing the applied voltage in increments of 0.5volts to 7volts. Record the data from each trial in your data sheet.
- Toggle “Output Enable” to “OFF” on the power supply console and show your measurements to the TA.

**Equation 1: Circuit Voltage Equation**

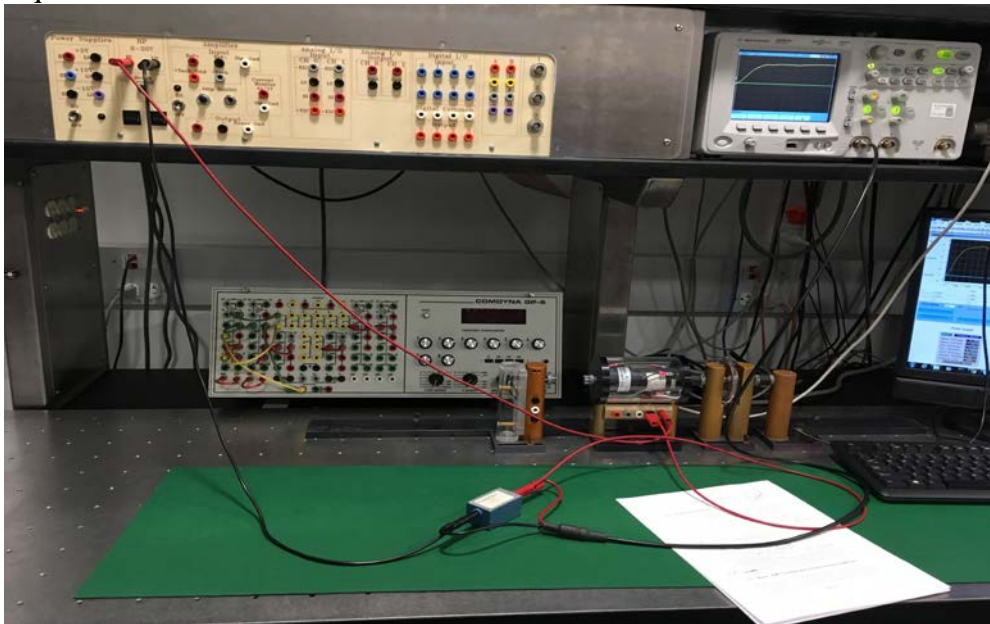
$$e_a(t) = i_a(t)R_a + L_a \frac{di_a(t)}{dt} + e_b(t)$$

**Equation 2: Angular Velocity - Back EMF**

$$e_b(t) = K_b \frac{d\theta(t)}{dt}$$

## 1.6 Armature Inductance

Consider again Equation 1. By locking the rotor (Figure 3) we will once again reduce the back EMF to zero. However, we are now concerned with finding the armature inductance and hence will consider the transient response of the resulting armature circuit. By putting a “Resistor Box” in series with the motor we can then measure the voltage drop across the resistor to graphically obtain the time constant. Once the time constant is found Equation 3 can be solved for the motor’s inductance.



**Equation 3: Time constant for RL circuit**

$$\tau = \frac{L_a}{R_a + R_s}$$

If you are not clear where this equation comes from, ask your TA for further explanation.

- Measure the resistance  $R_s$  of the resistance box and verify your value with the TA.
- Open the file: “n:/labs/ge320/exp3/la.vxe”
- Set volts per division of channel 1 of the scope to 0.6 V/div. If after performing the below steps you cannot see the transient response with enough resolution, adjust the **Time Base** of the scope smaller to enlarge the response on the scope. Click on **Send New Scope Parameters** after changing values.
- Reset the power supply and turn the “output enable” switch “OFF”. (You must do this EVERY time you open a new file).
- Set the current to 2.5 amps and the voltage to 5V.
- Connect the resistor box in series with the motor and scope the voltage drop across the resistor box.
- Toggle the power supplies output to “ON” and view the scope’s plot. Collect the data into the VEE program and then toggle the power supplies output to “OFF.” Estimate the time constant of the transient response (IT SHOULD BE IN MILLISECONDS). Use this value of  $\tau$  along with  $R_a$  and  $R_s$  to compute  $L_a$  using Equation 3. Record the values in your data sheet.

**1.7 Back – EMF Constant and Viscous Friction Coefficient**

The goal of this section will be to compute the back – EMF constant ( $K_b$ ). To do so, the motor will be permitted to spin freely and measurements will be taken when the system has reached its steady state. Consider Equation 1. Once steady state is reached,  $I_a$  will be constant, therefore, its derivative will be zero. Substituting into Equation 1 yields Equation 4:

**Equation 4**

$$E_a = I_a R_a + K_b \omega$$

The angular velocity ( $\omega$ ) can be found by measuring the tachometer voltage and using the gain found in Lab 1. Since  $K_b = K_T$  (in the SI system, as you proved in the prelab), finding  $K_b$  will give  $K_T$ . At steady state conditions, combining Equation 5 and Equation 6 yields Equation 7, which allows us to compute the viscous Friction Coefficient (B).

**Equation 5: Torque – current Equation**

$$T_m = K_T i_a(t)$$

**Equation 6: Mechanical equation**

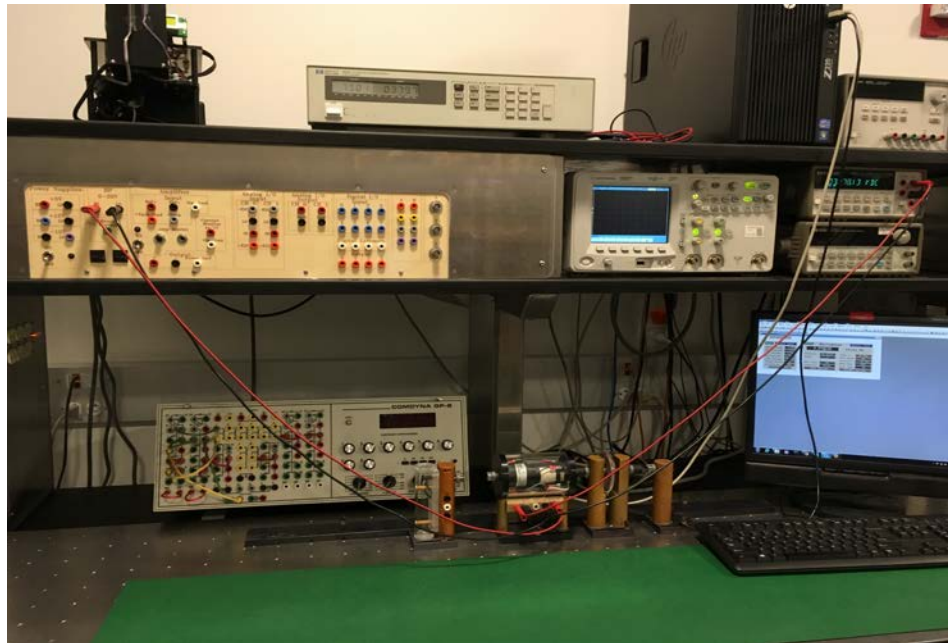
$$T_m = J \frac{d^2 \theta(t)}{dt^2} + B \frac{d \theta(t)}{dt}, \frac{d \theta(t)}{dt} = \omega$$

**Equation 7**

$$K_T I_a = B \omega$$

Follow the directions below:

- Remove the rotor–locking attachment and the resistor box. Connect the power supply directly to the motor.
- Use the DMM (Digital Multimeter) to measure the tachometer voltage. The orange lead of the tachometer is the positive terminal and the gray lead is the negative terminal.

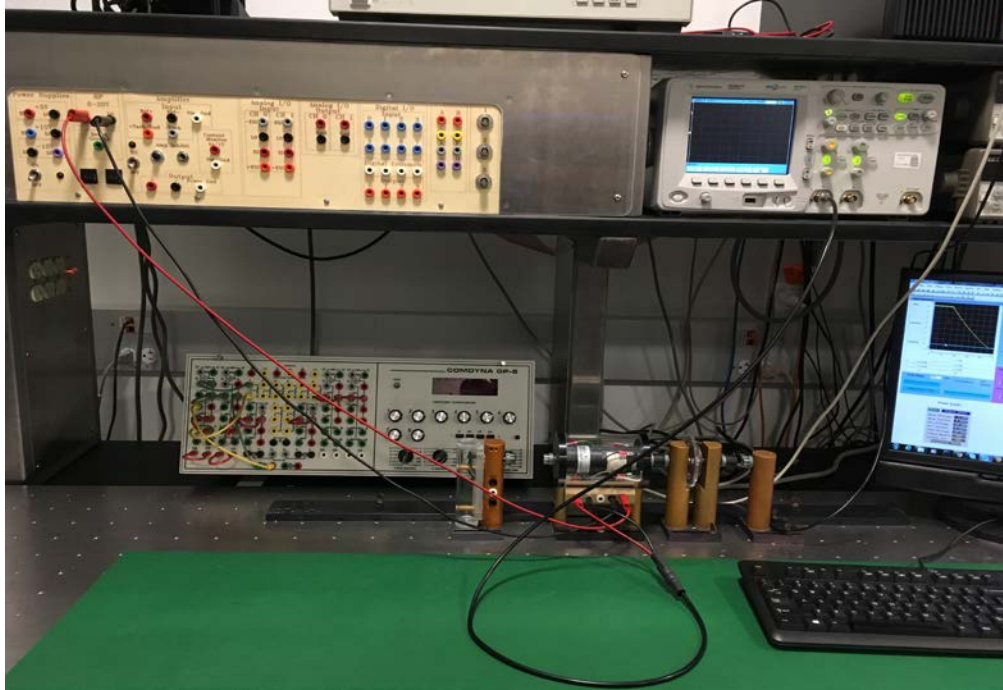


- Open the Agilent VEE file: “n:/labs/ge320/exp3/kb-n-b.vxe”
- From the computer “Reset” the power supply and then switch “Output Enable” off. Set the current to 2.5 amps and the voltage to 7.5V.
- Switch “Output Enable” to on and give the system some time to reach its steady state (a couple of seconds will be more than sufficient). Measure the applied voltage ( $E_a$ ), the steady–state current ( $I_a$ ) drawn by the motor, and the voltage generated by the tachometer.
- Turn off the power supply using the “Output Enable” button.
- Compute angular velocity ( $\omega$ ) and parameters  $K_b$ ,  $K_T$ , and  $B$  and record these values.
- Repeat the experiment with 10 volts applied to the motor.

## 1.8 Rotor Moment of Inertia

The motor's moment of inertia ( $J$ ) and the friction coefficient ( $B$ ) establish the angular velocity decay rate once current to the motor has been cut. Equation 6 links  $J$  and  $B$  to the time constant of the velocity decay since the  $T_m = 0$ . With  $B$  known, we can find  $J$ . ( $\tau = J/B$ ). Unplugging the High voltage from patch panel will cut the current instantaneously. If you are not clear where  $\tau = J/B$  came from, ask your TA for further explanation.

- Scope the tachometer's output voltage using channel 1 on the scope.



- Open the file “`n:/labs/ge320/exp3/j.vxe.`” Reset the power supply; toggle its output to “OFF”, set its current to 2.5 amps and voltage to 15V. Set the scope's **Time Base** to 200ms.
- Toggle the power supply's output to “ON” and wait a second for the motor to reach steady state. Disconnect the High voltage connection at HP variable supply from patch panel and watch the voltage display on the scope. Quickly press the “Collect Data from Scope” button in VEE to store the waveform, which should be an exponential decay.
- Estimate the time-constant  $\tau_m$ . Using the values of  $B$  and  $\tau_m$  calculate  $J$ .
- Make sure that all values are recorded in your data sheet.

### 1.9 Lab 2: Post Lab

Include answers to the following questions in your lab report. Be brief.

1. Using the various quantities measured in this experiment, compute the voltage to angular velocity second order transfer function  $\omega(s)/E_a(s)$ .
2. Theoretically, in the experiment for the measurement of  $J$  the angular velocity of the rotor should decay exponentially. Do you think the decay looked exponential? Explain your observation.
3. Compute the poles of the second order transfer function  $\omega(s)/E_a(s)$ . Justify, in terms of speed of the poles, that a first order transfer function can be used to model this motor. Derive the first order transfer function  $\omega(s)/E_a(s)$ .
4. What is the motor's voltage to angular **position** transfer function  $\theta(s)/E_a(s)$ ?

### 1.10 Data Sheet

Armature Resistance		
Current	Voltage	R <sub>a</sub>
Average		

Armature Inductance	
Resistor Box resistance (R <sub>s</sub> )	
Time Constant ( $\tau$ )	
Armature Inductance (L)	

Back – EMF Constant and Viscous Friction Coefficient					
E <sub>a</sub>	I <sub>a</sub>	V <sub>tach</sub>	$\omega$	K <sub>b</sub> (=K <sub>t</sub> )	B
7.5V					
10V					

Rotor Moment of Inertia	
Time Constant ( $\tau$ )	
Moment of Inertia (J)	