

## 4 Experiment 4: DC Motor Voltage-to-Speed Transfer Function Estimation by Step-Response and Frequency-Response (Part 2)

### 4.1 Introduction

This lab introduces new methods for estimating the transfer function of a plant (in our case, the DC motor). Recall that in Experiment 3 the transfer function of the motor was obtained by measuring the various physical parameters of the motor and applying them to the known mathematical model. The methods to be used in this lab, called step- and frequency-response methods, are indirect and can be used even if a good mathematical model of the plant is unavailable.

### 4.2 Pre-lab

1. Consider  $A \times \sin(\omega t)$  as the input to a linear system with Transfer Function  $G(s)$ . Write down the expression for the output of the system.
2. Determine the equations of the sine waves depicted in Figure 1. The y-axis is magnitude and the x-axis is time ranging from 0 to  $2\pi$  seconds. The larger amplitude signal is the input and the smaller amplitude signal is the output.
3. Sketch the Bode diagram, both magnitude and phase, for the system  $\frac{K_m}{s\tau_m + 1}$
4. Using the Bode plot from the previous question (#3), determine the DC gain and time constant for the system. Mark them clearly on your graph.

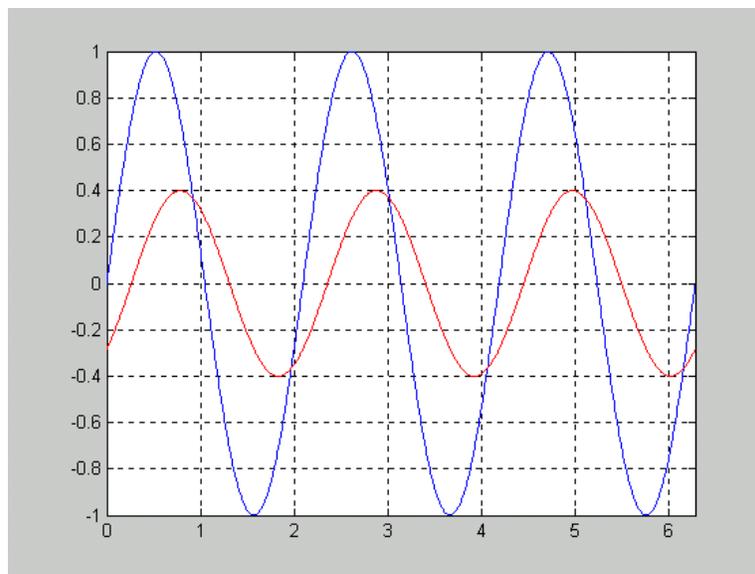


Figure 1 – Graph of input and output Sinusoids

### 4.3 Objectives

- Create a Bode plot from measured frequency data.
- Determine the transfer function from the frequency response.

### 4.4 References

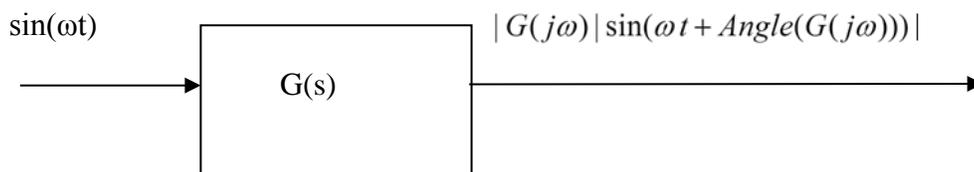
*Feedback Control Systems* (2011), Fifth ed. by Charles L. Phillips and John M. Parr.

Note: the same information is in *Feedback Control Systems* (2000), Fourth ed. by Charles L. Phillips and Royce D. Harbor.

- For Step Response: see Chapter 4
- For Frequency Response & Bode Plots: see Chapter 8

### 4.5 Transfer Function Estimation by Frequency Response

A very powerful tool for identifying systems is the *frequency response method*. The key idea behind frequency response techniques is to input sinusoids into the linear system and study the sinusoidal response. A well-known result of linear theory establishes that the steady-state response of a linear system to a sinusoid is a sinusoid, with the same frequency as the input sinusoid, but, possibly, with a different amplitude and phase. The change in amplitude is equal to  $|G(j\omega)|$  and the phase shift is  $Angle(G(j\omega))$ , where  $G(s)$  is the transfer function of the system, evaluated at the input frequency. Figure 2 shows a graphical representation of this idea.

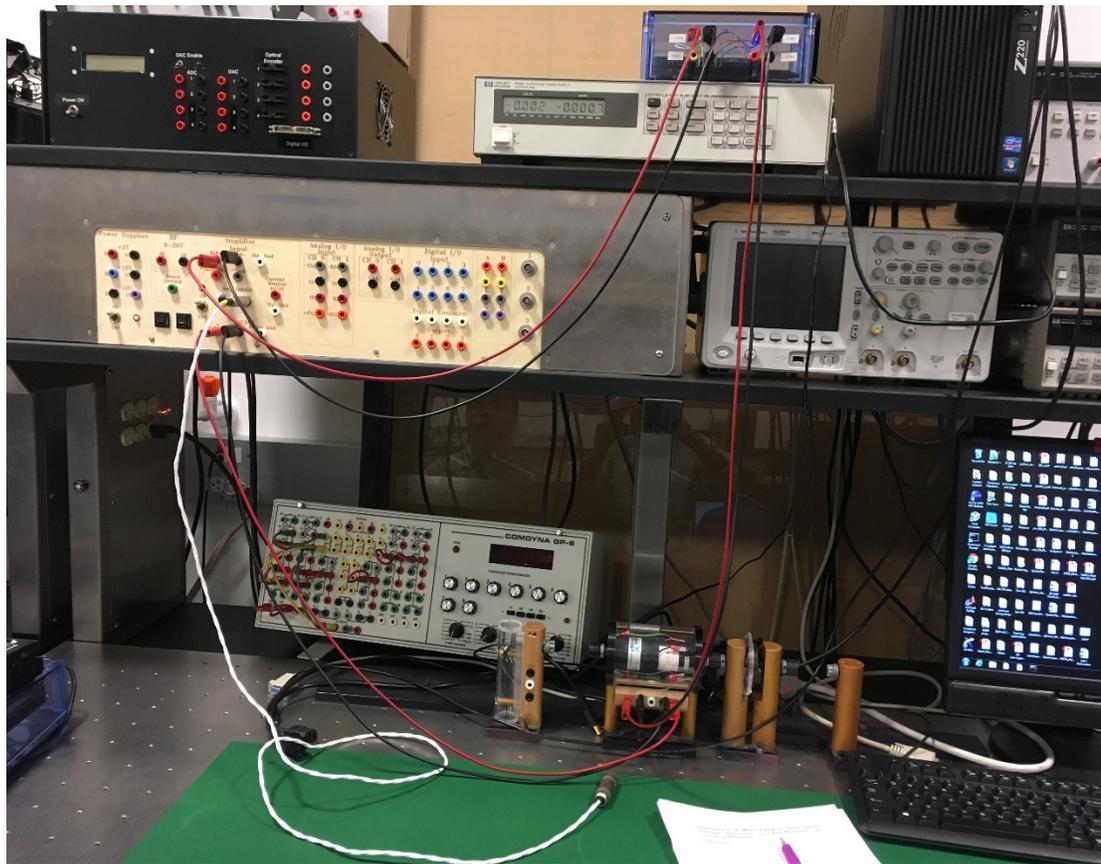


**Figure 2 - Steady-State response of a linear system to a sinusoid**

The goal of this section will be to input sine waves of varying frequencies and measure the ratio of amplitudes from output to input sine waves and the phase shift between the two sinusoids. This will provide enough information to identify the linear system  $G(s)$ , ignoring static (coulombic) friction. The input wave needs to have a DC offset of sufficient magnitude (Why?). When calculating the ratio between output and input amplitudes, you will have to eliminate the DC offsets both for the input and output signals. An easy way to do this is to compare the peak to peak voltages instead of the amplitudes. *From your prelab you should have a good idea on how to use this information to obtain the transfer function of your system. If you still have doubts, ask your TA.*

## Testing Procedure:

- Set up the myRIO, patch panel amplifier, and motor in the same way that was done for part 1 of the lab. The output of the myRIO should go to the input of the amplifier, and the output of the amplifier should be connected to the input of the motor. Connect the tachometer to the analog 1 input of the myRIO. Lastly, insert the thumb switch to the quarter inch audio jack on the amplifier.



- Open LabVIEW and select **Open Existing**. Open the “Lab4.lvproj” project file from the folder you created during the first part of the lab. After this, the project explorer window should appear.
- From the Project explorer window open the starter file “Freq\_response.vi”. The Front Panel will appear. Press “ctrl+E” to open the Block diagram window.
- Add a **Sine Signal** block and a **Discrete Transfer Function** blocks.
- The **Discrete Transfer Function** can be found in **Control & Simulation** -> **Simulation** -> **Discrete Linear Systems**. Double click on it and set the parameters to match Figure 9. This helps reduce the noise that you will see on the tachometer signal.
- Connect the new blocks to match Figure 3.
- Double click on the **Sine Signal** block to set the parameters. Set the amplitude to **1**, and the offset to **3**. You will have to repeat the experiment for the frequencies 0.1Hz, 0.5Hz, 1Hz, 2Hz, 3Hz, 4Hz and 5Hz. **Note**: for frequencies of 3 – 5 Hz

disconnect the filter from the XY Graph Buffer and hook up the Analog input directly.

- You need to identify the interval at which a  $45^\circ$  phase – shift occurs (Why?). To find the  $45^\circ$  point more accurately, pick three more frequencies within that interval and find the gain and phase shift for those frequencies.
- Change the length of the simulation in the Control & Simulation loop parameters to 20 seconds.
- Press the thumb switch and keep it pressed while the program is running on the myRIO.

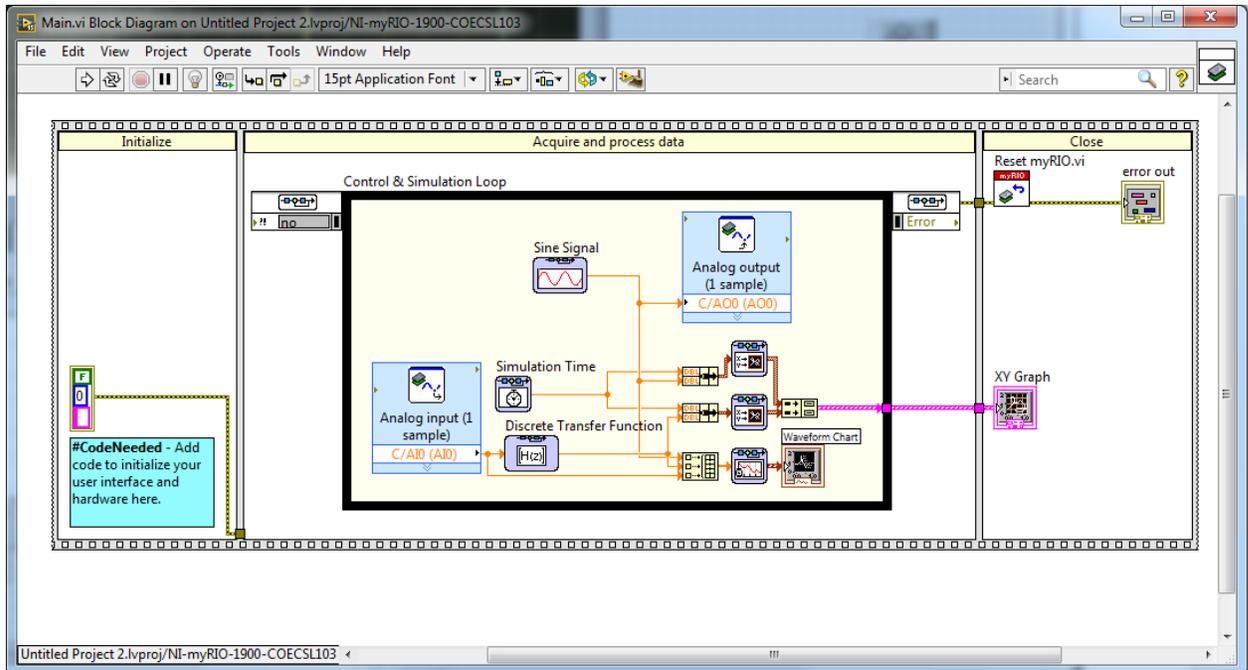
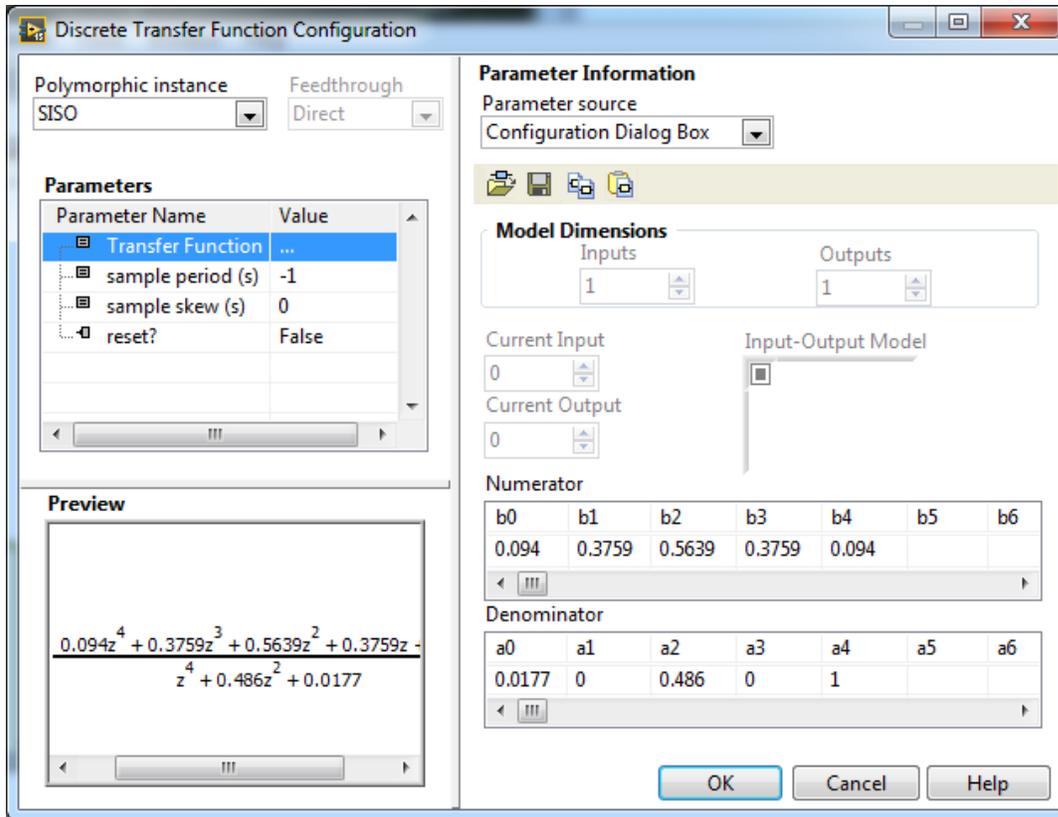


Figure 3 - Block Diagram window for Frequency Response



**Figure 4 - Discrete Transfer Function Configuration Window**

- To create the magnitude plot, find the ratio of peak-to-peak output voltage verses peak-to-peak input voltage. This is called the “gain” at that particular frequency. When creating your Bode plot, it is common to convert this “gain” to units of decibels (dB)

$$1 \text{ dB} = 20 \log_{10} K$$

- To measure the phase shift, use two adjacent peaks of the input and output sinusoids and measure the time difference between them. The output wave is said to “lag,” or be “behind” the input wave. You need to convert this time measurement to degrees. Hint: Remember that 1 period of the wave is  $360^\circ$  (period =  $1/\text{frequency}$ ).
- Repeat for the frequencies indicated. As you increase the frequency, you can change the length of the simulation in the Control & Simulation loop parameters. You don’t need to change it for each repetition of the experiment but make sure that you have at least five full periods of the wave on the scope’s screen.
- Create a Bode plot with the recorded data. You will need to generate a semilog plot of magnitude (in dB) verses input frequency and a semilog plot of phase-shift (in degrees) verses input frequency. You can use Excel or the command “semilogx” in MATLAB to create a log plot, i.e. semilogx(freq,gaindB).
- From these two plots find values for  $K_m$  and  $\tau_m$ . (Post Lab Question #1)

## **4.6 Post Lab**

Include the answers to the following questions in you lab report.

1. Calculate the transfer function for the motor with the data obtained from the frequency–response method.
2. In the frequency–response method, why did we need a DC offset?
3. How can we infer the order of the system from the Bode plot of the frequency-response method?

### 4.7 Data Sheet

#### FREQUENCY RESPONSE OF A DC MOTOR

Frequency (Hz)	V <sub>in</sub> p-p (V)	V <sub>out</sub> p-p (V)	$\frac{V_{outpp}}{V_{inpp}}$ gain	Gain in dB	Δt (sec)	Phase shift (degrees)
0.1						
0.5						
1						
2						
3						
4						
5						
Extra measurements for 45° phase shift						