Laboratory handouts, ME 340

This document contains summary theory, solved exercises, prelab assignments, lab instructions, and report assignments for Lab 3.

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Laboratory handout 3 – Block diagrams and simulation

Given the derivative \( \dot{x}(t) := \frac{dx(t)}{dt} \) of the signal \( x(t) \), we recover \( x(t) \) by integration:

\[
x(t) = x(0) + \int_0^t \dot{x}(\tau) \, d\tau.
\]

This relationship is represented in the following block diagram.

\[
\begin{array}{c}
\dot{x}(t) \\
\hline
x(0) + \int_0^t \\
x(t)
\end{array}
\]

Given the differential equation

\[
\dot{x}(t) + \frac{1}{T} x(t) = f(t),
\]

it follows that

\[
\dot{x}(t) = f(t) - \frac{1}{T} x(t).
\]

The following partial block diagram shows the dependence of the derivative \( \dot{x}(t) \) on \( x(t) \) and the input \( f(t) \).

A complete block diagram is obtained by combining the partial block diagram with the representation of the relationship between \( \dot{x}(t) \) and \( x(t) \):
Given the differential equation

\[ m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t), \tag{4} \]

it follows that

\[ \ddot{x}(t) = \frac{1}{m}(f(t) - c\dot{x}(t) - kx(t)), \tag{5} \]

where \( \ddot{x}(t) = \frac{d^2x(t)}{dt^2} \). The following partial block diagram shows the dependence of the derivative \( \ddot{x}(t) \) on \( x(t) \), \( \dot{x}(t) \), and the input \( f(t) \).

Since

\[ \dot{x}(t) = \dot{x}(0) + \int_0^t \ddot{x}(\tau) \, d\tau \tag{6} \]

or, equivalently,

the following block diagram provides a complete representation of the original differential equation.

In \textsc{matlab}, the \textsc{simulink} environment provides support for
graphical construction of block diagrams and simulation of the resulting dynamical system. To start simulink, enter `simulink` on the command line:

```matlab
>> simulink
```

When the Simulink Library Browser window is open, enter `<ctrl>+n` on your keyboard to open a new model window.

To build the single-component block diagram

```
<table>
<thead>
<tr>
<th>sin(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 + ∫₀ᵗ</td>
</tr>
<tr>
<td>x(t)</td>
</tr>
</tbody>
</table>
```

open the Continuous Library. Select the Integrator icon and enter `<ctrl>+i` on your keyboard to add this component to your model.

Next, open the Sources Library. Select the Sine Wave icon and enter `<ctrl>+i` on your keyboard to add this component to your model. Finally, open the Sinks Library. Select the Scope icon and enter `<ctrl>+i` on your keyboard to add this component to your model. Click and drag each of the components in the model window to arrange them on the model canvas.

Connect the components sequentially by clicking on the appropriate output port and dragging the marker to the appropriate input port. The complete model should look like this:

```
Sine Wave → 1/s → Scope
```

Finally, to set the properties of each component, double-click on each icon and assign the appropriate parameters, for example the initial value 2 for the integrator. Note that double-clicking on the Scope opens up a separate window that can be used to view the output of the Integrator.

To simulate the output \(x(t)\) to the input \(\sin t\), enter `<ctrl>+t` on your keyboard. The Scope shows a graph of the function

\[
2 + \int_0^t \sin \tau \, d\tau = 3 - \cos t.
\]
Click on the Autoscale icon to fit the graph to the Scope window.

Let \( u(t) = 1 \) for \( t \geq 0 \) and 0 otherwise. To build a Simulink model representing the following block diagram

\[
2u(t) + \left( -1 + \int_{0}^{t} u(t) \, dt \right) + \frac{1}{2} x(t) = x(t)
\]

focus on the Simulink Library Browser window, and enter <ctrl>+n on your keyboard to open a new model window and save this to disk. Add an Integrator from the Continuous Library, a Gain and a Sum from the Math Operations Library, a Step from the Sources Library, and a Scope from the Sinks Library, and arrange these on the model canvas. You can flip the Gain component horizontally by selecting its icon on the canvas and entering <ctrl>+i on your keyboard.

To set the properties of the components, double-click on each component and enter the appropriate settings. For the Sum, enter |+- in the “List of signs” field.

Finally, connect the components sequentially by clicking on the appropriate output port and dragging the marker to the appropriate input port. You can split off a separate connection to the Scope from the connection between the Integrator and the Gain by right-clicking at some point on the corresponding wire and dragging the marker to the input port on the Scope.

To save the output of a simulation to the MATLAB workspace, add a To Workspace component from the Sinks Library to your model and connect it appropriately. Double-click on its icon to set the variable name for the output, e.g., “xout”. Set the “Save Format” to “Array”. Once the simulation is complete, you can use the MATLAB plot command to graph the time history, e.g.,

\[
>> \text{plot}(\text{tout}, \text{xout})
\]

The complete model should look like this:
Provided that the signal \( f(t) \) is bounded by some exponential function for all \( t \geq 0 \) in the time domain, then its Laplace transform

\[
\mathcal{L}[f(t)](s) := \int_0^\infty f(t)e^{-st} \, dt
\]

is defined for all complex numbers \( s \) in some right half plane in the frequency domain.

When both sides are defined,

\[
\mathcal{L}[\alpha f(t)](s) := \alpha \mathcal{L}[f(t)](s),
\]

\[
\mathcal{L}[f(t) + g(t)](s) := \mathcal{L}[f(t)](s) + \mathcal{L}[g(t)](s),
\]

and

\[
\mathcal{L}\left[ \int_0^t f(\tau) \, d\tau \right](s) := \frac{1}{s} \mathcal{L}[f(t)](s).
\]

When the relationship between the Laplace transforms \( X(s) \) and \( Y(s) \) is of the form of a product

\[
Y(s) = H(s) \cdot X(s)
\]

for some transfer function \( H(s) \), given by the Laplace transform of the corresponding unit impulse response \( h(t) \), then \( y(t) \) is given by the convolution \( (h(\#) * x(\#))(t) \).

Provided that \( x(0) = 0 \), the time-domain block diagram
\[
\int_{t_0}^{t} f(t) \, dt = x(t)
\]
is equivalent to the frequency-domain block diagram

![Frequency-domain block diagram 1](image1)

where \( F(s) := \mathcal{L}[f(t)](s) \) and \( X(s) := \mathcal{L}[x(t)](s) \). This implies that

\[
X(s) = \frac{1}{s} \left( F(s) - \frac{1}{T} X(s) \right) \Rightarrow X(s) = \frac{T}{sT + 1} F(s)
\]

in terms of the transfer function \( H(s) = \frac{T}{sT + 1} \), or, equivalently,

![Transfer function block diagram](image2)

Similarly, provided that \( x(0) = 0 \) and \( \dot{x}(0) = 0 \), the time-domain

![Time-domain block diagram](image3)

is equivalent to the frequency-domain block diagram

![Frequency-domain block diagram 2](image4)
where \( F(s) := \mathcal{L}[f(t)](s) \) and \( X(s) := \mathcal{L}[x(t)](s) \). This implies that

\[
X(s) = \frac{1}{ms^2} \left(F(s) - csX(s) - kX(s)\right) \Rightarrow X(s) = \frac{1}{ms^2 + cs + k} F(s) \quad (14)
\]

in terms of the transfer function \( H(s) = 1/(ms^2 + cs + k) \), or, equivalently,

\[
F(s) \xrightarrow{\frac{1}{ms^2 + cs + k}} X(s)
\]

When \( x(0) \neq 0 \), the time-domain block diagram

\[
f(t) + \int_0^t f(t) \, dt + x(0) \Rightarrow x(t)
\]

is equivalent to the frequency-domain block diagram

\[
F(s) + \frac{1}{s} + \frac{1}{T} \Rightarrow X(s)
\]

where \( F(s) := \mathcal{L}[f(t)](s) \) and \( X(s) := \mathcal{L}[x(t)](s) \). This implies that

\[
X(s) = \frac{1}{s} \left(x(0) + F(s) - \frac{1}{T} X(s)\right) \Rightarrow X(s) = \frac{T(x(0) + F(s))}{sT + 1} \quad (15)
\]

The **free response** when \( f(t) = 0 \) for all \( t \) is then equal to the unit impulse response corresponding to the transfer function

\[
\frac{T x(0)}{sT + 1}. \quad (16)
\]

In **MATLAB**, a transfer function that is the ratio of two polynomials in \( s \) may be constructed using the `tf` command. The command

\[
\text{tf}(num, den)
\]
>> sys=tf(2,[2 1])

assigns a representation of the transfer function

\[ H(s) = \frac{2}{2s + 1} \]  

(17)

to the variable sys. A plot of the corresponding unit impulse response is obtained using the impulse command:

>> impulse(sys)

Similarly, if \( L[x(t)](s) = H(s) \cdot L[f(t)](s) \) and \( f(t) = 1 \) for all \( t \geq 0 \), then a plot of the corresponding unit step response \( x(t) \) is obtained using the step command:

>> step(sys)

When the relationship between the Laplace transforms of the input and output to a dynamical system can be expressed in terms of a transfer function \( H(s) \) that is the ratio of two polynomials in \( s \), then the corresponding block diagram can be constructed in Simulink by adding a Transfer Fcn component from the Continuous Library to a model.

The Simulink model corresponding to the block diagram

\[ \begin{align*}
F(s) & \quad \frac{3}{3s+1} \quad X(s)
\end{align*} \]

may then given by

where the numerator and denominator coefficients entered in the corresponding fields of the Transfer Fcn component are 3 and [3 1], respectively. Note that the input and output signals to the Transfer Fcn component are functions in the time domain. The frequency-domain transfer function represents a time-domain convolution.
Exercises

1. Draw a time-domain block diagram with input $f(t)$ and output $y(t)$ representing the differential equation

   \[
   \dot{y}(t) = f(t).
   \]

2. Draw a time-domain block diagram with input $f(t)$ and output $x(t)$ representing the differential equation

   \[
   \ddot{x}(t) - x(t) = f(t).
   \]

3. Draw a frequency-domain block diagram with input $F(s) := \mathcal{L}[f(t)](s)$ and output $Y(s) := \mathcal{L}[y(t)](s)$ representing the differential equation

   \[
   \dot{y}(t) = f(t).
   \]

   Find an expression for $Y(s)$ in terms of $F(s)$, $y(0)$, and $\dot{y}(0)$.

4. Draw a time-domain block diagram with input $r(t)$ and output $x(t)$ representing the differential equation

   \[
   \dddot{x}(t) + \dot{x}(t) + x(t) = \dot{r}(t) + r(t)
   \]

   with arbitrary initial conditions $x(0)$, $\dot{x}(0)$.

5. Draw a frequency-domain block diagram with input $R(s) := \mathcal{L}[r(t)](s)$ and output $X(s) := \mathcal{L}[x(t)](s)$ representing the differential equation

   \[
   \dddot{x}(t) + \dot{x}(t) + x(t) = \dot{r}(t) + r(t)
   \]

   with initial conditions $x(0) = 0$, $\dot{x}(0) = 0$, and $r(0) = 0$. Find the corresponding transfer function.

6. Find a differential equation for $x(t)$ in terms of $f(t)$ corresponding to the block diagram.
and find the relationship between $x(0)$ and $\dot{x}(0)$ and the initial conditions $a$ and $b$.

---

**Solutions**

1. Here,

$$f(t) \rightarrow y(0) + \int_0^t y(t)$$

2. Here,

$$f(t) \rightarrow \dot{x}(0) + \int_0^t \rightarrow x(0) + \int_0^t x(t) \rightarrow x(t)$$

3. Here,

$$\dot{y}(0) \rightarrow \frac{1}{s} \rightarrow y(0) \rightarrow \frac{1}{s} \rightarrow F(s) \rightarrow Y(s)$$

This implies that

$$Y(s) = \frac{1}{s} \left( y(0) + \frac{1}{s} (\dot{y}(0) + F(s)) \right) = \frac{sy(0) + \dot{y}(0) + F(s)}{s^2}.$$ 

4. Suppose that $y(t)$ satisfies the differential equation

$$\ddot{y}(t) + \dot{y}(t) + y(t) = r(t)$$
and that \(x(t) = \dot{y}(t) + y(t)\), from which it follows that

\[
\ddot{x}(t) + \dot{x}(t) + x(t) = \dot{r}(t) + r(t).
\]

Since \(\ddot{y}(0) = r(0) - \dot{y}(0) - y(0)\), it follows that \(\ddot{y}(0) = x(0) + \dot{x}(0) - r(0)\) and \(y(0) = r(0) - \dot{x}(0)\). An equivalent block diagram is shown below.

5. From the previous exercise, \(y(0) = \dot{y}(0) = 0\), and consequently, 

\[
\begin{align*}
\frac{1}{2} & \quad \frac{1}{2} \\
R(s) & \quad X(s)
\end{align*}
\]

This implies that

\[
X(s) = \frac{s + 1}{s^2 + s + 1} R(s)
\]

and the corresponding transfer function equals

\[
H(s) = \frac{s + 1}{s^2 + s + 1}.
\]

6. Let \(y(t)\) represent the output of the bottom integrator. Then,

\[
y(t) = b + 2 \int_0^t (x(\tau) - y(\tau)) \, d\tau,
\]

and

\[
x(t) = a + \frac{1}{2} \int_0^t \left( f(\tau) - 2(x(\tau) - y(\tau)) \right) \, d\tau.
\]

In particular, \(x(0) = a\) and \(y(0) = b\).
Differentiation with respect to $t$ now yields

\[ \dot{y}(t) = 2x(t) - 2y(t), \quad \dot{x}(t) = \frac{1}{2} f(t) - x(t) + y(t) \]

In particular, $\dot{x}(0) = \frac{1}{2} f(0) - a + b$. Moreover,

\[ \ddot{x}(t) = \frac{1}{2} \dot{f}(t) - \dot{x}(t) + \dot{y}(t) = \frac{1}{2} \dot{f}(t) + f(t) - 3\dot{x}(t) \]

or, in other words,

\[ \ddot{x}(t) + 3\dot{x}(t) = \frac{1}{2} \dot{f}(t) + f(t). \]
Prelab Assignments

Complete these assignments before the lab. Show all work for credit.

1. Consider the differential equation

\[ \ddot{x}(t) + 2\dot{x}(t) + 40x(t) = f(t). \]

(a) Draw a partial block diagram representing the dependence of the derivative \( \ddot{x}(t) \) on \( x(t) \), \( \dot{x}(t) \), and the input \( f(t) \).

(b) Draw a time-domain block diagram representation of the original differential equation in terms of integrators, amplifiers, summing junctions, and splitting junctions. Don’t forget the initial conditions. Label all connections between components to show the corresponding signals.

(c) Draw a frequency-domain block diagram representation for the case of 0 initial conditions and use this to express \( X(s) := \mathcal{L}[x(t)](s) \) in terms of \( F(s) := \mathcal{L}[f(t)](s) \) and a transfer function \( H(s) \).

(d) Draw a frequency-domain block diagram representation for the case of arbitrary initial conditions and use this to find an expression for \( X(s) := \mathcal{L}[x(t)](s) \) in terms of \( F(s) := \mathcal{L}[f(t)](s) \), \( x(0) \), and \( \dot{x}(0) \).

2. Consider the differential equation

\[ \frac{1}{6} \ddot{\theta}(t) + 2\dot{\theta}(t) + 9.8 \cos \theta(t) = 0 \]

(a) Draw a partial block diagram representing the dependence of the derivative \( \ddot{\theta}(t) \) on \( \theta(t) \) and \( \dot{\theta}(t) \).

(b) Draw a time-domain block diagram representation of the original differential equation in terms of integrators, amplifiers, summing junctions, splitting junctions, and a component representing the nonlinear relationship \( \theta \mapsto \cos \theta \). Don’t forget the initial conditions. Label all connections between components to show the corresponding signals.
3. Consider the differential equation

\[ 250 \ddot{x}(t) + 3000 \dot{x}(t) + 2000x(t) = 3000 \dot{r}(t) + 2000r(t) \]

(a) Let

\[ 250 \ddot{y}(t) + 3000 \dot{y}(t) + 2000y(t) = r(t) \]

and show that \( x(t) = 3000 \dot{y}(t) + 2000y(t) \) satisfies the governing differential equation.

(b) Assume that \( x(0) = \dot{x}(0) = r(0) = 0 \) and draw a time-domain block diagram representation of this differential equation in terms of integrators, amplifiers, summing junctions, and splitting junctions.

(c) Draw a corresponding frequency-domain block diagram representation, and use this to express \( X(s) := \mathcal{L}[x(#)](s) \) in terms of \( R(s) := \mathcal{L}[f(#)](s) \) and a transfer function \( H(s) \).

(d) Use the \texttt{tf} and \texttt{impulse} commands in \texttt{MATLAB} to graph the function whose Laplace transform equals \( H(s)/s \). This is the unit step response.
Lab instructions

In this lab you will learn how to use the SIMULINK environment in MATLAB to model and simulate dynamic systems using block diagrams. You can think of SIMULINK as a tool that allows programming in a graphical manner. Instead of large amounts of code, you can simply add pre-made components into a “model” window and connect the components’ inputs and outputs to create a system that can be simulated in SIMULINK. The progress of the simulation can be monitored while the simulation is running, and the final results can be made available in the MATLAB workspace when the simulation is complete.

The steps required in order to simulate a system using SIMULINK are listed below.

1. Write the governing differential equations of the system.

2. Create a partial block diagram representing the relationship between the highest derivative in these equations and the remaining terms.

3. Use relations between the inputs and outputs of the partial block diagram to created a full simulation model using only integrators, amplifiers, summing, and splitting junctions.

4. Convert your block diagram to a SIMULINK representation and assign appropriate parameters to all components.

5. Decide on simulation parameters and run the simulation.

6. Display and analyze the results.

To build a SIMULINK representation, you must first add individual components to your model from the SIMULINK component libraries. You can do this by opening a library, selecting the icon for the desired component, and either dragging the component into the model window, or entering <ctrl>+i on your keyboard.

The components needed for this lab are contained in the Continuous Library (Integrator), the Math Library (Gain and Sum), Source
Library (Step), and the Sinks Library (Scope and To Workspace). Each library can be opened by double-clicking on its icon in the Simulink Library Browser. Browse through each of these libraries to become acquainted with the available components. The function of each components should be obvious from its title. However, if you are unsure about a component’s function or its use, double-click on the component to open its dialog box and then select Help to get a detailed explanation of its use.

Input and output ports on individual components are represented by angle brackets “>” pointing into or away from the component, respectively. To connect two components, click on the output port, drag the marker to the corresponding input port, and release the mouse button. A line with an arrowhead should now appear showing the connection. To split a connection, right-click on the desired location of the splitting junction, drag the new connection to the appropriate input port, and release the mouse button.

A connection or component can be deleted in Simulink by highlighting it and hitting the <delete> key on your keyboard. To change the position of a component, you can simply drag it from one location to another and its connections will remain intact. Finally, if you want to change the size of a component for readability, select the component and drag any corner until it is the desired size.

**A mechanical suspension**

Consider the motion of the mechanical suspension shown below, able to translate along the direction described by the vector \( \hat{i} \).
The displacement of the 1 kg mass is governed by the differential equation
\[ \ddot{x}(t) + 2\dot{x}(t) + 40x(t) = f(t). \] (18)

If \( X(s) := \mathcal{L}[x(t)](s) \) and \( F(s) := \mathcal{L}[f(t)](s) \), then
\[ X(s) = \frac{(s + 2)x(0) + \dot{x}(0) + F(s)}{s^2 + 2s + 40}. \] (19)

1. Refer to the time-domain block diagram found in the prelab assignment. Construct a **Simulink** representation of this dynamic system by opening a new **Simulink** model window, adding the corresponding components, and connecting these appropriately. Use a **Constant** component from the Sources Library to represent a constant \( f(t) \). Note that the orientation of the **Gain** components may be reversed by selecting the corresponding icons and entering <ctrl>+i on the keyboard. Double-click on each **Gain** component to enter the corresponding numerical constant.

2. Add a **Scope** component and a **To Workspace** component to monitor \( x(t) \) during simulation and store the corresponding time history to disk, respectively. Double-click on the **To Workspace** component and select “Array” in the “Save Format” drop-down menu. Enter “position” in the “Variable name” field, and 2000 in the “Limit data points to last” field.

3. Free response:
   
   (a) Enter initial conditions for the two integrators corresponding
to an initial position of 0.1 m and an initial velocity of 0 m/s.
Enter 0 for the value of the constant input. Enter <ctrl>+s on
your keyboard to save your SIMULINK model to the c:\matlab\me340
directory.

(b) Double-click on the Scope and run a simulation by entering
<ctrl>+t on your keyboard. Click on the “Autoscale” icon to fit
the graph to the window. In the MATLAB command window,
enter

```matlab
>> time_ic=tout;
>> pos_ic=position;
>> plot(time_ic, pos_ic)
```

to save the sequence of time steps and the position time history
to the MATLAB variables time_ic and pos_ic, respectively, and
to graph $x(t)$.

(c) In the MATLAB command window, use the tf command to
construct a transfer function equal to the right hand side of (19)
when $F(s) = 0$, $x(0) = 0.1$, and $x'(0) = 0$, and store the result in
the MATLAB variable sys. Enter

```matlab
>> [pos_imp,time_imp] = impulse(sys, 10);
```

to generate and graph the corresponding free response, and
to store the resulting sequence of time steps and positions in
time_imp and pos_imp, respectively.

4. Step response:

(a) Substitute a Step component from the Sources Library for the
Constant component representing the input. Double-click on the
Step component, enter 0 in the “Step time” and “Initial value”
fields, and enter 20 in the “Final value” field.

(b) Enter 0 for the initial conditions of the two integrators and run
a simulation by entering <ctrl>+t on your keyboard.

5. Read Report Assignment 1. and produce all the plots before you
go on to the next section.
A nonlinear pendulum

Consider the motion of the slender rod of length $\ell = 25$ cm and mass $M = 8$ kg shown below, able to rotate about a horizontal axis $\alpha$ that is perpendicular to the rod, under the influence of a vertical gravitational field of acceleration $g = 9.8$ m/s$^2$ and a viscous damping torque proportional to the angular velocity $\dot{\theta}(t)$ with proportionality constant $b = 0.2$ Nms.

The orientation $\theta(t)$ of the rod is governed by the differential equation

$$\frac{M\ell^2\ddot{\theta}(t)}{3} + b\dot{\theta}(t) + \frac{Mg\ell\cos\theta(t)}{2} = 0 \quad \text{(20)}$$

This equation is nonlinear because of the $\cos\theta(t)$ term. Such a nonlinearity may be represented in simulink using a Trigonometric Function component from the Math Library.

1. Refer to the time-domain block diagram found in the prelab assignment. Open the simulink representation of the mechanical suspension with constant input equal to 0, and select the “Save As...” menu item from the “File” menu to save a copy to disk with a new name. Add a Trigonometric Function component from the Math Library and double-click on this component to select the appropriate function. Modify the connections between the components to represent the nonlinear pendulum. Double-click on each Gain component to enter the corresponding numerical constant.

2. Enter 0 for the initial conditions of the two integrators, enter
<ctrl>+s on your keyboard to save your model, and run a simulation by entering <ctrl>+t on your keyboard.

3. For \( \theta(t) \approx -\pi/2 \), we obtain the linear approximation \( \cos \theta(t) \approx \pi/2 + \theta(t) \). The governing equation becomes

\[
\frac{M\ell^2\ddot{\theta}(t)}{3} + b\dot{\theta}(t) + \frac{Mg\ell}{2}\theta(t) = -\frac{Mg\pi\ell}{4}. \tag{21}
\]

Make the appropriate modifications to your Simulink model and select the “Save As...” menu item from the “File” menu to save to disk with a new name. Simulate this linear approximation with different initial conditions and compare to the nonlinear simulation with same initial conditions. At what initial conditions does the linear approximation become poor? Note: you may want to store the results of your simulations to different MATLAB variables or use the hold command with the plot command.

**A quarter-car model**

Consider the quarter-car vehicle-suspension model shown below, where the mass \( m = 250 \text{ kg} \) represents 1/4 of the mass of a car body, and the spring with stiffness \( k = 2000 \text{ N/m} \) and damper with damping coefficient \( b = 3000 \text{ Ns/m} \) represent a suspension spring and shock absorber, respectively. The position \( x(t) \) of the car body equals 0 when the road input \( r(t) \) equals 0 and the spring supports the weight of the body.

The vertical displacement of the body is governed by the differential equation

\[
m\ddot{x}(t) + b\dot{x}(t) + kx(t) = br(t) + kr(t) \tag{22}
\]
With $x(0) = \dot{x}(0) = r(0) = 0$, the relationship between the ground input $r(t)$ and the displacement $x(t)$ is described by the transfer function

$$H(s) = \frac{bs + k}{ms^2 + bs + k} \quad (23)$$

1. Construct a Simulink model using a Transfer Fcn component from the Continuous Library and double-click on this component to enter the polynomial coefficients for the numerator and denominator of $H(s)$. Let the input signal be a step of size 0.3 m and simulate the corresponding dynamic system.

2. Use two To Workspace components to store the input and output time histories to the MATLAB workspace and plot the difference $z(t) = x(t) - r(t)$. 
Report Assignments

Complete these assignments during the lab. Show all work for credit.

1. In the analysis of the mechanical suspension:

   (a) Use the MATLAB subplot command to graph the free response using the Simulink output with i) the variable-step ode45 integrator with relative tolerance $10^{-3}$; ii) the fixed-step integrator ode3 with automatic step-size; and iii) the fixed-step integrator ode3 with step size 0.1; as well as using the output of the impulse command. You can modify the simulation tolerances by putting focus on the Simulink model, entering <ctrl>+e on your keyboard, and adjusting the “Solver options” entries. Give the plot the title “Plot 1: Simulation and Analytical Response of a Mass-Spring-Damper System”, and label the x and y axes. Explain the observed differences.

   (b) Use the MATLAB hold command to plot several step responses in the same graph. Use the Simulink output with different values in the “Final value” field of the Step component.

2. In the analysis of the nonlinear pendulum:

   (a) Use the MATLAB hold command to plot the solution of the nonlinear and linearized models for initial conditions where the linear approximation is invalid. Give the plot the title “Plot 2: Linear and Nonlinear Simulation in Nonlinear Range”, and include a legend, as well as labels for the x and y axes.

   (b) Propose a range of initial conditions for which the linear approximation is reasonable. Justify your answer.

   (c) A nonlinear model is typically more accurate than the linear model, but also more costly to implement. What engineering considerations might determine whether a linear approximation is appropriate?

3. In the analysis of the quarter-car model:
(a) Plot the suspension travel $z(t)$ when the input is a 0.3 m step. Give the plot the title “Plot 3: Suspension Travel of the Quarter-Body Model to a 30 cm Curb”, and label the $x$ and $y$ axes.

(b) Plot the body position $x(t)$ when the input is a 0.3 m step. Give the plot the title “Plot 4: Quarter-Body Position Response to a 30 cm Curb”, and label the $x$ and $y$ axes.

(c) Suppose that you were designing the suspension so that the passengers of the vehicle would be protected from the effects of the car hitting the curb. What dynamic information would you want to obtain from the quarter-body simulation so that you could tell if a person could be hurt by the collision with the curb? For example, which signal in your block diagram would provide the dynamical information that is most closely related to an injury? Explain your answer.