

ME 360: FUNDAMENTALS OF SIGNAL PROCESSING, INSTRUMENTATION AND CONTROL

Experiment No. 5

System Identification with Frequency Response Techniques using the Dynamic Signal Analyzer

1. CREDITS

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2. OBJECTIVES

- (a) Manually vary the frequency input to obtain the gain and phase-shift (Bode) plots of a motor-generator system.
- (b) Become familiar with the functionality and operation of a dynamic signal analyzer.
- (c) Use a dynamic signal analyzer to obtain the gain and phase-shift (Bode) plots of a motor-generator system.
- (d) Use a dynamic signal analyzer to obtain the gain and phase-shift (Bode) plots of a beam-mass system.

3. KEY CONCEPTS

- (a) A Bode plot of a system can be found by varying the frequency of a sine wave input to the system and measuring the gain and phase-shift between this input and the system output signal.
- (b) A dynamic signal analyzer is a dedicated instrument that determines the frequency spectrum of an arbitrary waveform using Fast Fourier Transform (FFT) techniques.
- (c) A dynamic signal analyzer can also determine the gain and phase shift (Bode) plots for a basic input-output system using either a sinusoidal input (sine-sweep method) or a noise input.

4. SYNOPSIS OF PROCEDURE

- (a) A Bode plot for a motor-generator system is manually found by applying different frequency sine waves to the system and measuring the gain and phase-shift of the output.
- (b) Bode plots for a motor-generator system are obtained with the dynamic signal analyzer using both the sine-swept and white-noise methods. The steady-state gain and time constant of the system are determined from the plots.
- (c) Bode plots for a beam-mass system are obtained with the dynamic signal analyzer using the sine-swept method. The resonant frequency is compared with the theoretical value calculated from beam and mass parameters. This part of the procedure is carried out as a demonstration by the Laboratory Assistant for the group as a whole.

5. PROCEDURE

5.1 Manual Sine Sweep Identification

To get a good understanding of what the DSA is capable of automating for you, you will first manually generate a Bode plot of the frequency response of the motor-generator system. This is done by applying a number of different frequency sine waves to the system and measuring the gain and phase shift change between the system's input and output on the oscilloscope.

Wire the System

- (a) Connect the function generator's output to the amplifier input on the patch panel.
- (b) Connect the amplifier's output to the motor's plus, minus and ground jacks.
- (c) Connect oscilloscope Channel 1 to the amplifier input.
- (d) Connect oscilloscope Channel 2 to the motor's tachometer output.
- (e) Connect the push button (Amp Inhibit button) to the amplifier.

Program Function Generator

- (f) Program the function generator to produce a 0.2 Hz sine wave with amplitude of 2 V_{p-p} and 4-V DC offset.

Start Oscilloscope

- (g) Run the oscilloscope in roll mode under “Main/Delayed”. The tachometer signal is very noisy, so we would like to filter this. Press “Acquire” and select “High Resolution” to apply an averaging filter. Then start the motor by depressing the amplifier push button. Observe the input and output waveforms on the oscilloscope. Make sure you see only two or three periods of the input and output on the oscilloscope so you can accurately measure the gain and phase-shift.

Measure the peak-to-peak voltage of *channel one* and record this on the datasheet as $V_{p-p, in}$. Before moving on, make sure that the set point voltage and measured input voltage are in close agreement.

Determine Frequencies for Constructing Bode Plots

- (h) Slowly sweep the frequency between 0.2 and 10 Hz using the dial on the function generator. Notice how the gain and phase shift change with frequency.

Determine Gain $G(f)$ and Phase Shift $\phi(f)$ at Each Test Frequency Using Oscilloscope Cursors

- (j) Turn the oscilloscope cursors on and *select Channel 2 for the cursor source*. Use the cursors to measure $V_{p-p, out}$, the peak-to-peak voltage of the generator output. Record this value on the Data Sheet. Compute the gain at this frequency from

$$G(f) = \frac{V_{p-p, output}}{V_{p-p, input}}$$

Record $G(f)$ on the Data Sheet.

- (k) Measure the time delay τ_{delay} of the generator output voltage with respect to the motor input voltage using the time cursors. Record the time delay on the Data Sheet. Compute the phase shift at this frequency from

$$\phi(f) = -360^\circ \times f \tau_{delay}$$

Record $\phi(f)$ on the Data Sheet.

Construct Bode Plots Based on Results of Sinusoidal Inputs

- (l) Use the commands below to plot the bode plots. The values of the K and τ , determined from a graphical analysis of the Bode plots as described in the Appendix should be entered on the Data Sheet as $K_{mansweep}$, $\tau_{mansweep, -3dB}$, and $\tau_{mansweep, -45^\circ}$.

MATLAB Command	Action
<code>clear all</code>	clear all variables
<code>fs = [x.x x.x x.x x.x x.x x.x x.x x.x]</code>	enter the test frequencies into an array
<code>Gs = [x.xx x.xx x.xx x.xx ... x.xx]</code>	enter gains into an array
<code>phis = [xx.x xx.x xx.x xx.x ... xx.x]</code>	enter phase shifts into an array
<code>Gdbs = 20 * log10(Gs)</code>	convert gains to decibels [dB]
<code>semilogx (fs, Gdbs)</code>	plot gain in dB vs. frequency in Hz with frequency on a log scale
<code>xlabel ('f [Hz]')</code>	label x axis
<code>ylabel ('G [dB]')</code>	label y axis; print out gain plot
<code>semilogx (fs, phis)</code>	plot phase shift in degrees [°] vs. frequency in Hz with frequency on a log scale
<code>xlabel ('f [Hz]')</code>	label x axis
<code>ylabel ('phi [deg]')</code>	label y axis; print out phase-shift plot

5.2 Frequency Response of Motor-generator System

In this part of the experiment, we perform a frequency analysis of the motor-generator system using both a sinusoidal and a white-noise input. The frequency response captured by the signal analyzer will be used to determine the steady-state gain K and the time constant τ . The analyzer serves as the source of both the sinusoidal and white-noise inputs to the motor-generator. The output connector for the source is located on the rear of the analyzer.

Because this is your first experience with the signal analyzer, detailed programming instructions are given for this instrument. Here, we use the following conventions for specifying keystrokes.

- (a) Bold text is used to denote hard keys. To aid in locating the proper key, the control section (system, marker, display, or measurement) and the letter at the lower right-hand corner of the key (if one exists) is given in parentheses. For example, "**Inst Mode** (measurement J)" means press the Instrument Mode key, which is also the "J" key in the "Measurement" area of the signal analyzer controls.
- (b) Regular text is used to denote a function selected using one of the context sensitive keys labeled "F1" through "F9". The corresponding labels appear on the screen of the signal analyzer next to the function key, and the function key designation may also be shown in parentheses following the text. For example, "FFT Analysis (F1)" means press function key F1. In some cases, the function key toggles between several different settings. If so, the desired setting follows the function key separated by an equals sign. Thus, "AUTO CAL = OFF (F1)" means that you should press function key F1 until automatic calibration is turned off as indicated by the word "OFF" being highlighted on the screen.
- (c) Values entered using the numerical keypad are shown in *italics*. Often these values are followed by a units designator shown in regular text. The units are selected and the numerical value entered by pressing the corresponding context-sensitive function key. When no units are needed, the function key labeled "ENTER" is pressed instead of a units designator.

5.2.1 Configure the System

Connect Signal Analyzer Source Output to Analyzer Channel 1 and Power Amplifier Input

- (a) Locate the BNC patch cable connected to the source output on the rear of the signal analyzer. Connect the free end to the tee on Channel 1 of the signal analyzer.
- (b) Locate a patch cable with a BNC connector on one end and a pair of banana plugs on the other end.
- (c) Connect the cable between the tee on Channel 1 of the signal analyzer and the inputs of the power amplifier. Follow the red-black color code for the banana plugs.

Connect Power Amplifier Output to Motor-generator Input

- (d) Locate a gray, shielded patch cable with three banana plugs on each end.
- (e) Connect the output of the power amplifier to the input of the motor-generator assembly. Red is positive or high, black is negative or low, and white is ground. Follow the red-black-white color code on each end of the cable. You should have this completed in the previous section.

Connect Motor-generator Output to Analyzer Input Channel 2

- (f) Locate a patch cable with a BNC connector on one end and a pair of banana plugs on the other end.
- (g) Connect the motor-generator output to the tee on Channel 2 of the analyzer. On the motor-generator assembly, the orange socket is the positive generator output, and the gray socket is the negative generator output. Thus, the red banana plug should be plugged into the orange socket, and the black banana plug should be plugged into the gray socket.

Connect Oscilloscope Channel 1 to Power Amplifier Input

- (h) Locate another patch cable with a BNC connector on one end and a pair of banana plugs on the other end.
- (i) Connect oscilloscope Channel 1 to the input of the power amplifier. Follow the red-black color code of the banana plugs.

Connect Oscilloscope Channel 2 to Analyzer Channel 2

- (j) Connect oscilloscope Channel 2 to generator's output.

5.2.2 Program the Oscilloscope

- (a) Adjust the controls of the oscilloscope so that both Channel 1 and Channel 2 are displayed. Put the oscilloscope in roll mode ("Main/Delayed" hard key followed by "Roll" soft key). Set the time base to 500 ms/div and the vertical scale to 1 V/div. Move the reference for both channels to the first grid division from the bottom of the screen.

5.2.3 Program the Signal Analyzer for the Sine-swept Method

Keystrokes	Action
General System Programming	
Local/HP-IB Preset (system far left) DO PRESET (F1) System Utility (system V) CALIBRATN (F2) AUTO CAL = OFF (F1)	Initialize the signal analyzer
Set Up the Basic Measurement	
Inst Mode (measurement J) SWEPT SINE (F4)	Select frequency response determination using the sine swept method. In this method, the system is exposed to a series of sinusoidal waveforms, the gain and phase shift of the output signal is determined for each separate frequency, and the frequency-response plotted on the screen of the analyzer.
Freq (measurement K) START (F3) 0.5 (keypad) Hz (F2) STOP (F4) 10 (keypad) Hz (F2) RESOLUTION SETUP (F9) AUTO RESOLUTION ON (F4) MIN RESOLUTION (F6) 101 (keypad) Points/Sweep (F1) MAX % CHANGE (F5) 5 (keypad) % (F1)	Set the frequency range of the sinusoidal inputs to a lower limit of 0.5 Hz and an upper limit of 10 Hz. Turn "autoresolution" on. Set the minimum number of frequencies in the sine sweep to 101. Force more than the minimum number of frequencies if either the magnitude or phase shift at two adjacent frequencies changes by more than 5 %.
Source (measurement O) LEVEL (F2) 2 (keypad) Vpk (F1) DC OFFSET (F5) 4 (keypad) V (F1)	Set the peak-to-peak amplitude to 2 V. Set the offset to 4 V.
Set Up the Data Display	
Meas Data (display B) FREQ RESP 2/1 (F5)	Measure frequency response based on ratio of Channel 2 (generator output) to Channel 1 (power amplifier/motor input)
Disp Format (display H) MORE (F9) BODE DIAGRAM (F2)	Select Bode plots of both gain and phase shift
Scale (display D) AUTOSCALE = ON (F1)	Turn on the "autoscale" function for the plot
Turn On Display Marker	
Marker (marker A) MARKER = ON (F1)	Turn on the display marker
Insert the Open Plug into the Amplifier Inhibit Socket on the Patch Panel Enabling the Amplifier and Allowing the Motor to Run	
When the " Start " button is pushed as shown below, the signal analyzer will excite the system with a series of sine waves at increasing frequencies. Observe the input and output waveforms on the screen of the oscilloscope. At low frequencies, the output tracks the input well. However, as frequency is increased the output begins to lag the input, and the amplitude of the output signal is reduced. Also, observe the Bode plots as they are constructed on the screen of the signal analyzer.	
Start (measurement R)	Start the sine-sweep process
Remove the Open Plug from the Amplifier Inhibit Socket on the Patch Panel Disabling the Amplifier and Preventing the Motor from Running	

5.2.4 Find Steady-state Gain K and Time Constant τ using Single Points on the Sine-swept Bode Plots

- (a) Using the display marker and the magnitude plot, pick off frequency-gain points to determine the steady-state gain K, the break frequency f_b , and the system time constant τ . Record your analysis on the Data Sheet.

Frequency Response of a First-order System:	Gain in dB:	Break Frequency:
$G(\omega) = \frac{K}{\sqrt{1 + (\tau\omega)^2}}$	$G[\text{db}] = 20 \log_{10}(G)$	$f_b = \frac{1}{2\pi\tau}$

5.2.5 Find Steady-state Gain K and Time Constant τ using a Fit to the Sine-swept Bode Plots

- (a) Download the frequency response data to the computer, and fit a first-order frequency response function to the data. This step is accomplished as follows.
- Launch the VEE Pro program by double clicking the appropriate icon on the Windows™ desktop.
 - Choose “Open” under the “File” menu. Move through the directory structure and open “dsaplot.vee” in “N:\Labs\ME360\dsaplot.vee”.
 - Click the yellow box labeled “Collect Data” to transfer the data from the DSA to the PC.
 - Click the yellow box labeled “Get Fit” to instruct the DSA to perform a transfer function fit to the data. In the dialog boxes generated by “Get Fit”, enter the lower and upper frequency limits of the sine sweep and the number of poles and zeros in the transfer function assumed for our model of the system.
- (b) Read the Gain and Pole values on the screen of the DSA, and record these values as K_{DSA} and p_{DSA} on the Data Sheet. Note that the form of the transfer function used by the DSA differs slightly from that used previously in our discussion of the motor-generator system. The two transfer functions are compared below.

K- τ Form of Transfer Function	Gain-Pole-Zero Form of Transfer Function Used by DSA
$\hat{G}(s) = \frac{K}{\tau s + 1}$	$\hat{G}(s) = \frac{2\pi K_{\text{DSA}}}{s - 2\pi p_{\text{DSA}}}$

Comparing these two forms, we see that

$$\tau = -\frac{1}{2\pi p_{\text{DSA}}} \quad \text{and} \quad K = -\frac{K_{\text{DSA}}}{p_{\text{DSA}}}$$

Use the above relationships to compute the time constant τ and steady-state gain K from the gain and pole values reported by the DSA. Record these values on the Data Sheet.

- (c) Press the **Local/HP-IB** (system X) key on the DSA to restore local control (i.e., control of the instrument using the keys on the front panel of the instrument).

5.2.6 Program the Signal Analyzer for the White-noise Method

This method is similar to the sine-swept method except that a white-noise input with ensemble averaging is used.

Keystrokes	Action
General System Programming	
Preset (system far left) DO PRESET (F1) System Utility (system V) CALIBRATN (F2) AUTO CAL = OFF (F1)	Initialize the signal analyzer
Set Up the Basic Measurement	
Inst Mode (measurement J) FFT ANALYSIS (F1)	Select FFT analysis
Freq (measurement K) START (F3) 0.5 (keypad) Hz (F2) STOP (F4) 10 (keypad) Hz (F2) RESOLUTION (LINES) (F9) 400 (keypad) Enter (F1)	Set the frequency range for the FFT to a lower limit of 0.5 Hz and an upper limit of 10 Hz. Set the resolution to 400 lines.
Source (measurement O) RANDOM NOISE (F5) LEVEL (F2) 2 (keypad) Vpk (F1) DC OFFSET (F3) 4 (keypad) V (F1) SOURCE = ON (F1)	Select a white-noise source. Set the peak-to-peak amplitude to 2 V. Set the offset to 4 V. Turn the source on.
Set Up the Data Display	
Meas Data (display B) FREQ RESP 2/1 (F6)	Select Bode plots of the frequency response based on the ratio of Channel 2 (generator output) to Channel 1 (amplifier/motor input)
Disp Format (display H) MORE (F9) BODE DIAGRAM (F2)	Select a Bode plot with both gain and phase shift displayed
Scale (display D) AUTOSCALE = ON (F1)	Turn on the "autoscale" function for the plot
Set Up Averaging and Collect Data	
Avg (measurement T) NUMBER AVERAGES (F2) 5 (keypad) ENTER (F1) AVERAGES = ON (F1)	Turn on ensemble averaging; set the number of trials to 5
Insert the Open Plug into the Amplifier Inhibit Socket on the Patch Panel Enabling the Amplifier and Allowing the Motor to Run	
When the "Start" button is pushed as shown below, the signal analyzer will excite the system with a white-noise signal. Observe the input and output waveforms on the screen of the oscilloscope. Also, observe how the Bode plots are constructed on the screen of the signal analyzer.	
Start (measurement R)	Start data collection; the analyzer will collect data and produce Bode gain and phase shift plots using the average data
Remove the Open Plug from the Amplifier Inhibit Socket on the Patch Panel Disabling the Amplifier and Preventing the Motor from Running	

5.2.7 Find Steady-state Gain K and Time Constant τ using Single Points on the Noise-input Bode Plots

(a) Repeat the procedure of Section 5.2.4 with the noise-input Bode plots.

5.2.8 Find Steady-state Gain K and Time Constant τ using a Fit to the Noise-input Bode Plots

(a) Repeat the procedure of Section 5.2.5 with the noise-input Bode plots.

5.3 Frequency Response of a Beam-mass System (TA Demonstration)

5.3.1 Configuring the System

- (a) Disconnect the two cables with BNC connectors on one end and push pins on the other.
- (b) Move the cart with the beam shaker apparatus into position in front of the laboratory bench. **DO NOT POWER THE APPARATUS AT THIS TIME.**
- (c) Using a BNC patch cable, connect the Source Output of the analyzer to the amplifier input.
- (d) Using a second BNC patch cable, connect the Force Transducer Output located on the vertical support of the beam apparatus to the tee on Channel 1 of the oscilloscope.
- (e) Using a third BNC patch cable, connect the Accelerometer Output located on the horizontal base of the beam apparatus to the tee on Channel 2 of the oscilloscope.
- (f) Make sure the power switch on the front of the amplifier is in the "off" position. Plug in the amplifier power strip to the power strip on the laboratory bench. Power on both power strips. The cooling fans of the amplifier should begin operating.

5.3.2 Programming the Oscilloscope

- (a) Change both Channel 1 and Channel 2 to AC coupling.

5.3.3 Programming the Signal Analyzer for the Sine-swept Method for the Beam / mass Apparatus

Keystrokes	Action
General System Programming	
Preset (system far left) DO PRESET (F1) System Utility (system V) CALIBRATN (F2) AUTO CAL = OFF (F1)	Initialize the signal analyzer
Load Previously Saved Instrument Setup Parameters from Flexible Diskette	
Save/Recall (system Z) CATALOG = ON (F8)	Make sure the flexible diskette with the previously saved instrument parameters in the drive of the instrument, then access the directory on the diskette using the disk utilities
Use the marker control knob to move the highlight bar to the file named "BEAM.STA"	
RECALL STATE (F5) ENTER (F1) CATALOG = OFF (F8)	Load the parameters from the flexible diskette; wait while data is read, then turn diskette directory display off
Start (measurement R)	TURN ON THE POWER AMPLIFIER using the front-panel switch, then start the sine sweep process; observe the source and output waveforms on the screen of the oscilloscope as the sweep process is occurring

- (a) When the "**Start**" button is pushed as shown above, the signal analyzer will excite the system with a series of sine waves at increasing frequencies. Observe the input and output waveforms on the screen of the oscilloscope. Also, observe how the Bode plots are constructed on the screen of the signal analyzer.

5.3.4 Determining the Natural Frequency of the System

- (a) Using the large knob in the Marker section of the analyzer controls, move the display marker to large peak in the frequency response curve characteristic of the beam-mass system. Read the frequency, gain and phase shift, and record these values on the Data Sheet.
- (b) Move the display marker to the secondary peak in the frequency response curve. This peak is characteristic of the cart on which the system rests. Read the frequency, gain and phase shift, and record these values on the Data Sheet.

APPENDIX A. SINUSOIDAL RESPONSE

Analysis of the sinusoidal response data of a first order system is a two-step process. The first step is to obtain the frequency response curves or Bode plots from the time-domain response data. The second step is to find K and τ from the Bode plots.

Step 1 – Construct Bode Plots

Method 1 involves the direct measurement of the input and output waveforms. We use the function generator to drive the system with a series of sine waves at different frequencies. At each frequency, we measure the following four quantities and compute the gain and phase lag as indicated below.

$$\Delta V_{in} = \text{peak-to-peak amplitude of } V_{in}(t) \qquad \Delta V_{out} = \text{peak-to-peak amplitude of } V_{out}(t)$$

$$t_{\text{delay}} = \text{time lag between } V_{in} \text{ and } V_{out} \text{ waveforms}$$

$$G(f) = \Delta V_{out} / \Delta V_{in} \qquad \phi(f) = 360^\circ f t_{\text{delay}}$$

Bode plots are made by plotting the gain in decibels against the frequency in Hz where the gain in decibels is given by

$$G [\text{dB}] = 20 \log_{10}(G)$$

Refer to Fig. A.1 for annotated Bode plots for a first-order system.

Step 2 – Obtain Dynamic Parameters

The frequency response of a first-order system is given by

$$G(f) = \frac{K}{\sqrt{1 + (2\pi f\tau)^2}} \qquad \text{and} \qquad \phi(f) = -\arctan(2\pi f\tau).$$

Thus, we have that

$$G(0) = K$$

$$G(f_b) = \frac{K}{\sqrt{2}} = K - 3 \text{ dB} \qquad \text{and} \qquad \phi(f_b) = -45^\circ \quad \text{when} \quad 2\pi f_b \tau = 1 .$$

The dynamic system parameters are obtained from the Bode plots as follows:

- (a) K is the asymptotic gain as $f \rightarrow 0$.
- (b) $\tau_G = 1 / (2\pi f_{bG})$ is an estimate of τ based on the gain curve, where $G(f_{bG})$ is -3 dB from the low frequency gain (DC gain).
- (c) $\tau_\phi = 1 / (2\pi f_{b\phi})$ is an estimate of τ based on the phase-shift curve, where $\phi(f_{b\phi}) = -45^\circ$.

τ_G and τ_ϕ may differ owing to modeling and experimental errors.

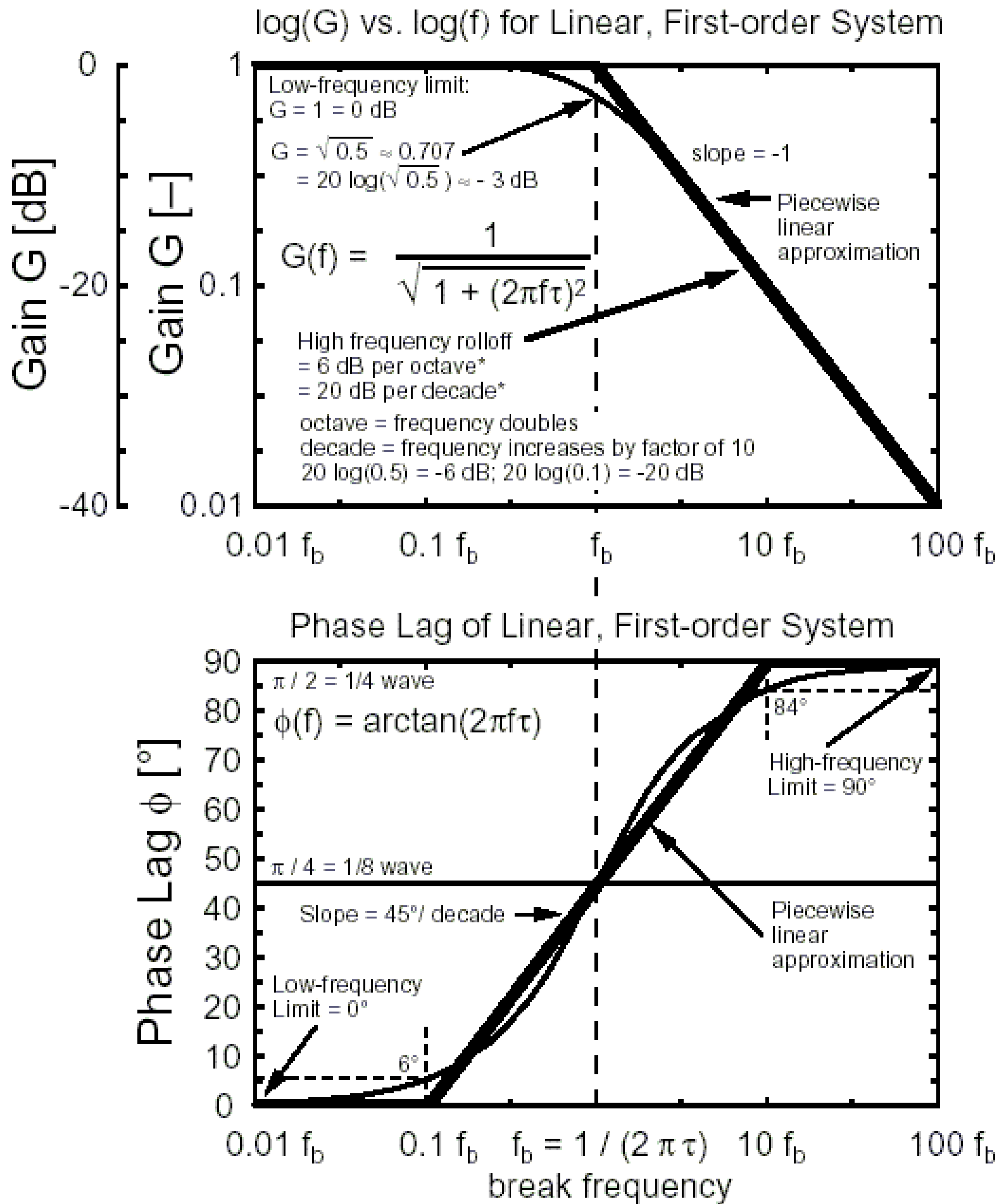


Figure A.1 Frequency Response Curves (Bode Plots) of a First-order Linear System.

Appendix B. Dynamic Signal Analyzer

The Dynamic Signal Analyzer (DSA) is a single device that possesses many of the combined capabilities of the oscilloscope, the function generator, the PC-based data acquisition hardware, and the MATLAB software. The primary difference lies in the user interface (instrument vs. computer) and speed (the DSA has a faster data capture rate). Also, the DSA provides front-end filtering and gain and offset adjustments that the PC-based system does not. These differences are consistent with the general observation that dedicated instruments are typically less flexible than PC-based methods in terms of being able to change the method, but instruments are typically easier to use and better at accomplishing the specific task for which they were designed. A block diagram of the digital signal analyzer is given in Fig. B.1.

In this experiment, we determine the frequency response (Bode) plots of two physical systems. The function generator built into the signal analyzer will be used to excite the systems. Both sinusoidal signals and white-noise signals are used for the motor-generator system studied in the last experiment. The results of the different methods are compared. The bode plot is used to determine the steady-state gain and time-constant of the motor-generator system. We shall also use the signal analyzer to determine the transfer function of a beam using sinusoidal inputs.

The controls of the signal analyzer are organized into six functional groups: (a) the power on/off and system functions located below the display, (b) the soft keys labeled F1-F9 with context sensitive functionality, (c) the marker controls used to operate the on-screen cursor, (d) the display controls for changing display scale, display coordinates, and other display parameters, (e) the numeric keypad for entering numeric values, and (f) the measurement controls that set basic measurement parameters. In addition, we note that the instrument has four input channels only two of which (1 and 2) are used in this experiment. Not shown in Figure A.2 but also used is the output of the built-in function generator known as the "source". This connector is on the rear of the instrument. The "source" indicator light on the front panel is lit when the function generator is in operation. We also note that the instrument has a standard 3.5-in. flexible disk for saving data and setup information. This disk can be used to transfer data to a PC-compatible computer. Alternatively, data can be transferred to an external device through the HP-IB (IEEE488) parallel interface available on the rear panel of the instrument.

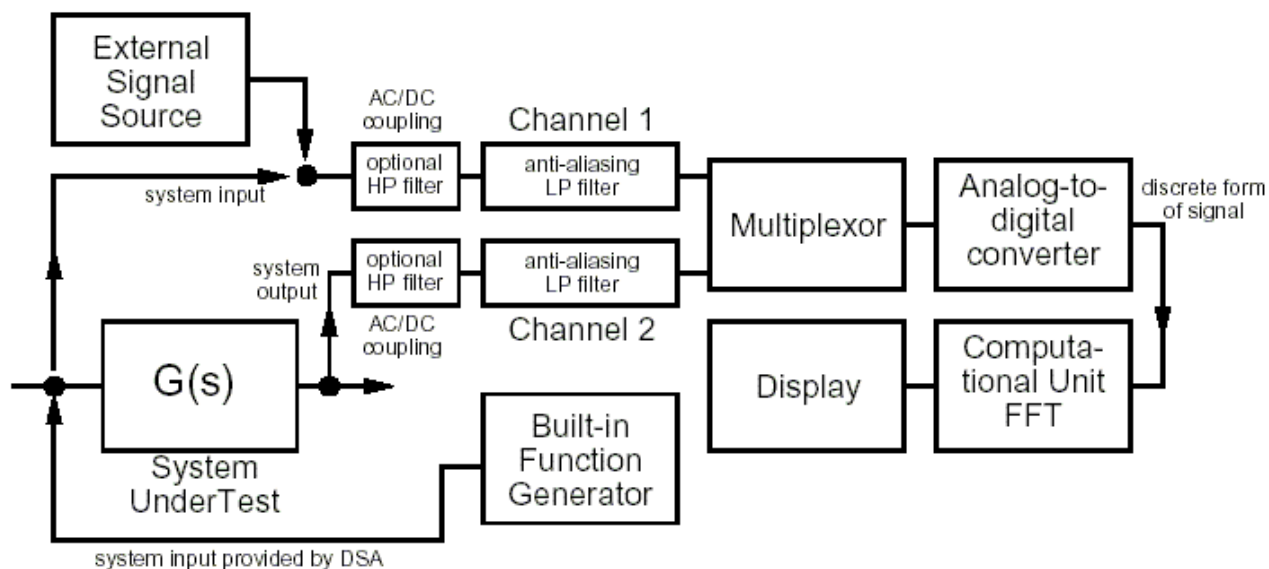


Figure B.1 Block Diagram of Digital Signal Analyzer.

We briefly consider each group of controls individually. The *marker controls* and *keypad* that operate in a straightforward manner and thus require no further discussion. The *system function* keys are used only briefly in this experiment to turn off autocalibration; this specific operation is also straightforward as set forth in the procedure, and, again, no further discussion is needed. The *measurement* and *display* keys work in conjunction with the soft keys to determine the type of measurement being made and how the results are displayed. Typical operation involves first pressing the appropriate hard key in the measurement or display group then selecting one or more of the context-sensitive soft key options. The measurement parameters should be set first; these parameters cannot be changed during a measurement. The display parameters, on the other hand, can be changed before, during, or after a measurement is made. In general, one begins in the upper left-hand corner of each group and works to the right and down through the various hard key functions. Specific programming instructions are given for each part of the experiment; however, you should understand the purpose of each setup procedure and not just blindly follow a set of prescribed keystrokes. You are also encouraged to try different options remembering that each change is permanent and immediately effective. Use discretion in choosing options to avoid moving so far from the prescribed operation as to distort the results or require extensive reprogramming of the instrument.

Appendix C. Frequency Response of a Beam-mass System

Here, we study a mechanical system consisting of a horizontal steel beam of rectangular cross section fixed at both ends and operated on at its midpoint by a large electromagnet (known as a shaker) as shown in Figure B.1. A concentrated mass is attached to the center of the beam. To properly analyze this system, we must consider the inertial (mass) contributions of the beam itself, the concentrated mass, and the moving element of the shaker.

A lumped-parameter (spring-mass-dashpot) analysis reveals several important aspects of the problem. Table C.1 summarizes the key equations and results of this analysis. The system is characterized by three parameters: (a) the mass M representing the total system inertia, (b) the spring constant K representing the elastic part of the system where energy is stored in a deformed member, and (c) the dashpot constant D representing the dissipation of kinetic energy into heat by friction. Unless the kinetic energy is very large as is the case for heavy objects or high speeds, the temperature rise due to dissipation may be so small as to be totally negligible. The dashpot may also be referred to as a viscous or inelastic effect. The three parameters M , D , and K collapse into the natural frequency $\omega_n = \sqrt{K / M}$ and the damping ratio $\zeta = D / (2 M \omega_n)$ which, in turn, become the characteristic parameters of the transient system. These parameters may be further restated in terms of the damped natural frequency ω_d and the resonance frequency ω_r .

To include the beam in the lumped-parameter analysis, we must consider its vibration modes as summarized in Table C.2. Table C.3 gives further information about the first four vibration modes. Although an infinite number of modes exist, we may assume that the fundamental mode dominates. We then use the static spring constant to determine an effective mass for the beam. Thus,

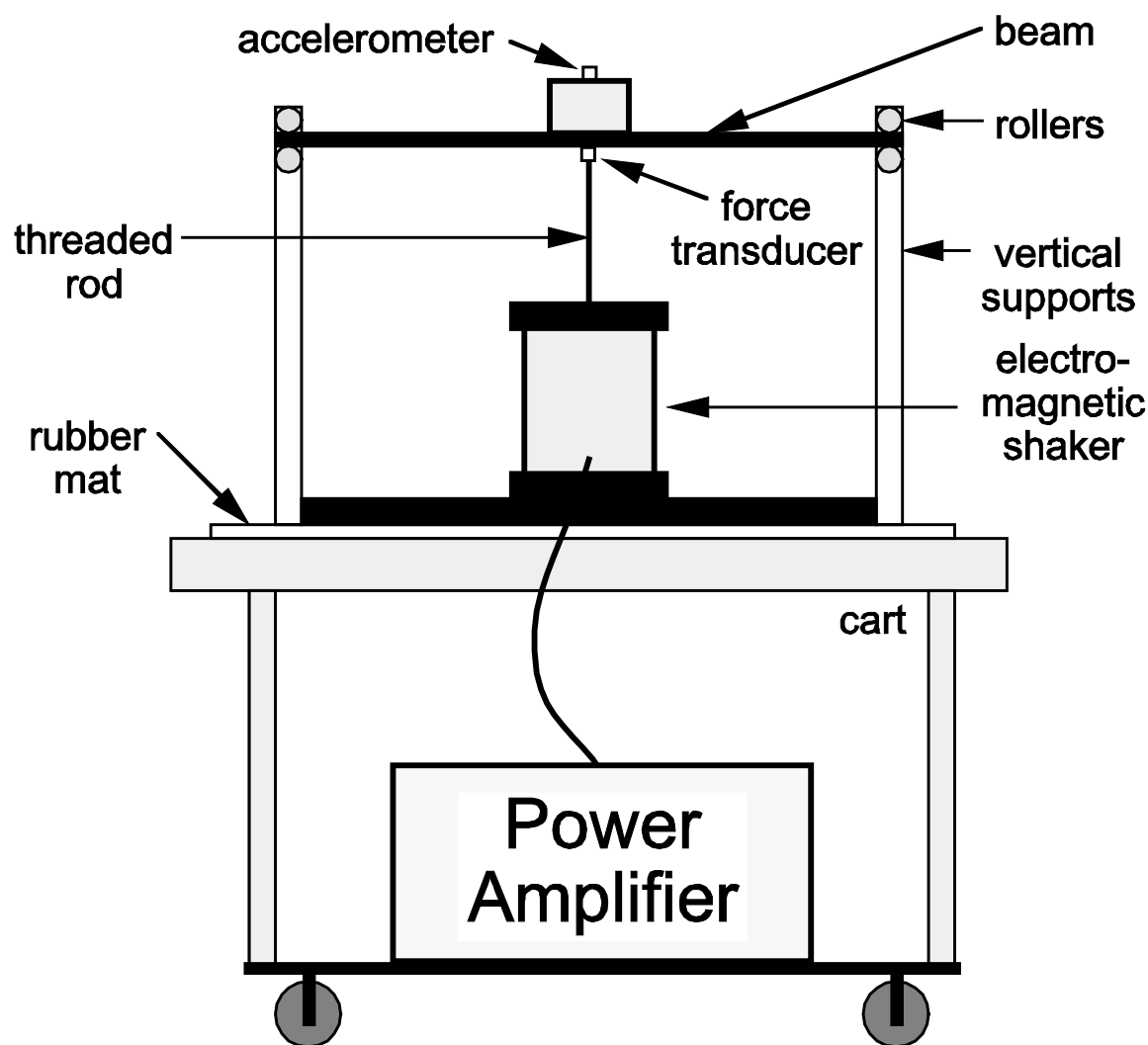


Figure C.1 Beam-mass system studied in this experiment.

Natural Frequency of Fundamental Mode from Beam Vibration Analysis

$$(\omega_{n,1})^2 = \frac{K}{m_{\text{eff}}} = (B_1)^2 \frac{EI}{m_{\text{beam}} L^3}$$

Static Spring Constant for Simple Beam Free at Both Ends

$$K = 48 \frac{EI}{L^3}$$

Effective Mass of Beam Based on Fundamental-mode Natural Frequency

$$m_{\text{eff}} = \frac{48}{(B_1)^2} (m_{\text{beam}}) = \frac{48}{(22.37)^2} (m_{\text{beam}}) = 0.09592 m_{\text{beam}}$$

Because the beam, concentrated mass, and shaker are all rigidly connected, the masses of these components may be summed and a new natural frequency computed for the system. Thus,

Total System Mass

$$M = m_{\text{eff}} + m_{\text{conc}} + m_{\text{shk}}$$

Natural Frequency Based on Total System Mass

$$\omega_n = \sqrt{\frac{K}{M}} = \omega_{n,1} \sqrt{\frac{m_{\text{eff}}}{M}}$$

Table C.1 Summary of lumped-parameter vibration analysis.

Governing Equation	$M \frac{d^2y}{dt^2} + D \frac{dy}{dt} + K y(t) = F(t)$
Parametric Form of Governing Equation	$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y(t) = f(t) = \frac{F(t)}{M}$ $\omega_n = \sqrt{\frac{K}{M}} \quad \zeta = \frac{D}{2M\omega_n}$
Displacement Frequency Response	$\tilde{G}_y(\omega) = \frac{1}{\omega_n^2 - \omega^2 + j 2\zeta\omega_n\omega}$ $G_y(\omega) = \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \quad \phi_y(\omega) = -\arctan(2\zeta\omega_n\omega, \omega_n^2 - \omega^2)$
Velocity Frequency Response	$\tilde{G}_v(\omega) = \frac{j\omega}{\omega_n^2 - \omega^2 + j 2\zeta\omega_n\omega}$ $G_v(\omega) = \frac{\omega}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \quad \phi_v(\omega) = -\arctan(\omega_n^2 - \omega^2, 2\zeta\omega_n\omega)$
Acceleration Frequency Response	$\tilde{G}_a(\omega) = \frac{-\omega^2}{\omega_n^2 - \omega^2 + j 2\zeta\omega_n\omega}$ $G_a(\omega) = \frac{\omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \quad \phi_a(\omega) = -\arctan(2\zeta\omega_n\omega, \omega_n^2 - \omega^2)$
Poles of the Transfer Function or Eigenvalues of the Coefficient Matrix	$\lambda_1 = \omega_n \left(-\zeta + \sqrt{\zeta^2 - 1} \right) \quad \lambda_2 = \omega_n \left(-\zeta - \sqrt{\zeta^2 - 1} \right)$
Damped Natural Frequency	$\omega_d = \omega_n \sqrt{1 - \zeta^2}$

Displacement Resonance	$\omega_{r,y} = \omega_n \sqrt{1 - 2\zeta^2}$	$G_y(\omega_{r,y}) = \frac{1}{2\zeta \sqrt{1 - \zeta^2}}$
Velocity Resonance	$\omega_{r,v} = \omega_n$	$G_v(\omega_{r,v}) = \frac{1}{2\zeta}$
Acceleration Resonance	$\omega_{r,a} = \frac{\omega_n}{\sqrt{1 - 2\zeta^2}}$	$G_a(\omega_{r,a}) = \frac{1}{2\zeta \sqrt{1 - \zeta^2}}$

The damped natural frequency ω_d is the frequency at which the system oscillates given an initial displacement and no forcing function. The amplitude of the oscillations die away exponentially with time constant $\tau = 1 / (\zeta \omega_n)$. The resonant frequencies are those frequencies at which the displacement, velocity, and acceleration gain curves reach a maximum.

Table C.2 Summary of beam vibration analysis.

Dimensional Governing Equation	$m' \frac{\partial^2 y}{\partial t^2} = -E I \frac{\partial^4 y}{\partial x^4} + F'(x, t)$
Dimensional Boundary Conditions	$y(0, t) = 0 \quad \left. \frac{\partial y}{\partial t} \right _{x=0} = 0 \quad y(L, t) = 0 \quad \left. \frac{\partial y}{\partial t} \right _{x=L} = 0$
Nondimensional Governing Equation	$\frac{\partial^2 y_*}{\partial t_*^2} = -\frac{\partial^4 y_*}{\partial x_*^4} + F_*(x_*, t_*)$
Nondimensional Boundary Conditions	$y_*(0, t_*) = 0 \quad \left. \frac{\partial y_*}{\partial t_*} \right _{x_*=0} = 0 \quad y_*(1, t_*) = 0 \quad \left. \frac{\partial y_*}{\partial t_*} \right _{x_*=1} = 0$
Definitions of Dimensional Variables	$x = \text{position} \quad t = \text{time} \quad y = \text{beam displacement}$ $m' = \text{beam mass per unit length} \quad F' = \text{distributed force per unit length}$ $E = \text{modulus of elasticity} \quad L = \text{beam length} \quad h = \text{beam thickness}$ $b = \text{beam width} \quad I = \text{moment of inertia of beam cross sectional area} = bh^3 / 12$
Definitions of Nondimensional Variables	$x_* = x / L \quad y_* = y / L \quad t_* = t \sqrt{\frac{E I}{m L^3}} \quad F_* = \frac{L^3 F'}{E I}$
Series Solution	$y_*(x_*, t_*) = \sum_{n=0}^{\infty} A_n \Psi_n(x_n) \exp(-j \omega_n t_*)$
Vibration Modes	$\Psi_n(x_*) = \sin(\lambda_n x_*) - \sinh(\lambda_n x_*) - D_n [\cos(\lambda_n x_*) - \cosh(\lambda_n x_*)]$ $D_n = \frac{\sin(\lambda_n) - \sinh(\lambda_n)}{\cos(\lambda_n) - \cosh(\lambda_n)}$
Vibration Frequencies	$\omega_n = B_n \sqrt{\frac{E I}{m L^3}} \quad f_n = C_n \sqrt{\frac{E I}{m L^3}} \quad B_n = (\lambda_n)^2 \quad C_n = \frac{(\lambda_n)^2}{2\pi}$
Eigenvalue Equation	$\cos(\lambda) \cosh(\lambda) = 1 \quad \lambda_n \text{ are roots; } \lambda_n \approx (2n + 1) \pi / 2$
Spring Constant	$K = 48 \frac{E I}{L^3}$ This form assumes that ends of beam are free and thus $\partial y / \partial x \neq 0$; if beam is clamped, K is 4 times greater

Figure C.2 is a block diagram illustrating the system and how it is tested. The DSA source drives the electromagnetic shaker that, in turn, applies a force to the beam-mass system. The output of the force transducer is routed to Input Channel 1 of the analyzer, and the output of the accelerometer is routed to Input Channel 2 of the analyzer. The sine-swept method is used to determine the frequency response of the system based on force transducer voltage as the system input and accelerometer voltage as the system output. The system transfer function in the Fourier Transform domain is thus given by

$$\tilde{V}_a(\omega) = \frac{-K_a \omega^2}{K_F M (\omega_n^2 - \omega^2 + j 2 \zeta \omega^n \omega)} \quad \tilde{V}_F(\omega) = \frac{-K_{a/F} \omega^2}{\omega_n^2 - \omega^2 + j 2 \zeta \omega^n \omega} \tilde{V}_F(\omega)$$

Figure C.3 shows the ideal frequency response of the system for the above transfer function, and $\omega_n = 21.5$ Hz, $K = 0.36$, and $\zeta = 0.054$. The natural frequency is that determined from the vibration analysis, and the other two parameters are based on experiment measurements. Note the low frequency asymptote has a -2 slope, and the high-frequency asymptote is flat.

Table C.3 Vibration mode parameters for simple beam fixed at both ends.

Mode	Coefficients	Modal Shape
1	$\lambda_1 = 4.730$ $B_1 = 22.37$ $C_1 = 3.561$ $D_1 = 1.018$	
2	$\lambda_2 = 7.853$ $B_2 = 61.67$ $C_2 = 9.816$ $D_2 = 0.9992$	
3	$\lambda_3 = 10.996$ $B_3 = 120.90$ $C_3 = 19.24$ $D_3 = 1$	
4	$\lambda_4 = 14.137$ $B_4 = 199.86$ $C_4 = 31.81$ $D_4 = 1$	

Table C.4 - Important Beam Parameters.

Parameter	Symbol and Definition	Value
Beam Length (between the vertical supports)	L	0.620 m
Beam Thickness (in the direction of the applied force)	h	0.00635 m
Beam Width (normal to the applied force)	b	0.0508 m
Beam Material	–	steel
Beam Modulus of Elasticity (steel)	E	2×10^{11} N / m ²
Beam Density (steel)	ρ	7860 kg / m ³
Moment of Inertia of Beam Cross Sectional Area	$I = bh^3 / 12$	1.084×10^9 m ⁴
Beam Mass	$m_{\text{beam}} = \rho b h L$	1.57 kg
Concentrated Mass	m_{conc}	1.76 kg
Reciprocating Mass of Shaker and Connecting Rod	m_{shk}	0.47 kg
Spring Constant of Beam	$K = 48 \frac{E I}{L^3}$	43700 N / m
Natural Frequency for First Mode of Vibration	$\omega_{n,1} = B_1 \sqrt{\frac{E I}{m_{\text{beam}} L^3}}$	538 rad / s
Effective Mass of Beam	$m_{\text{eff}} = K / (\omega_{n,1})^2$	0.15 kg
Total Lumped Parameter System Mass	$M = m_{\text{beam}} + m_{\text{conc}} + m_{\text{shk}}$	2.38 kg
Natural Frequency of Lumped Parameter System	$\omega_n = \sqrt{\frac{K}{m_{\text{total}}}}$	135 rad / s
Natural Frequency of Lumped Parameter System	$f_{n,1} = \frac{\omega_{n,1}}{2\pi}$	21.5 Hz

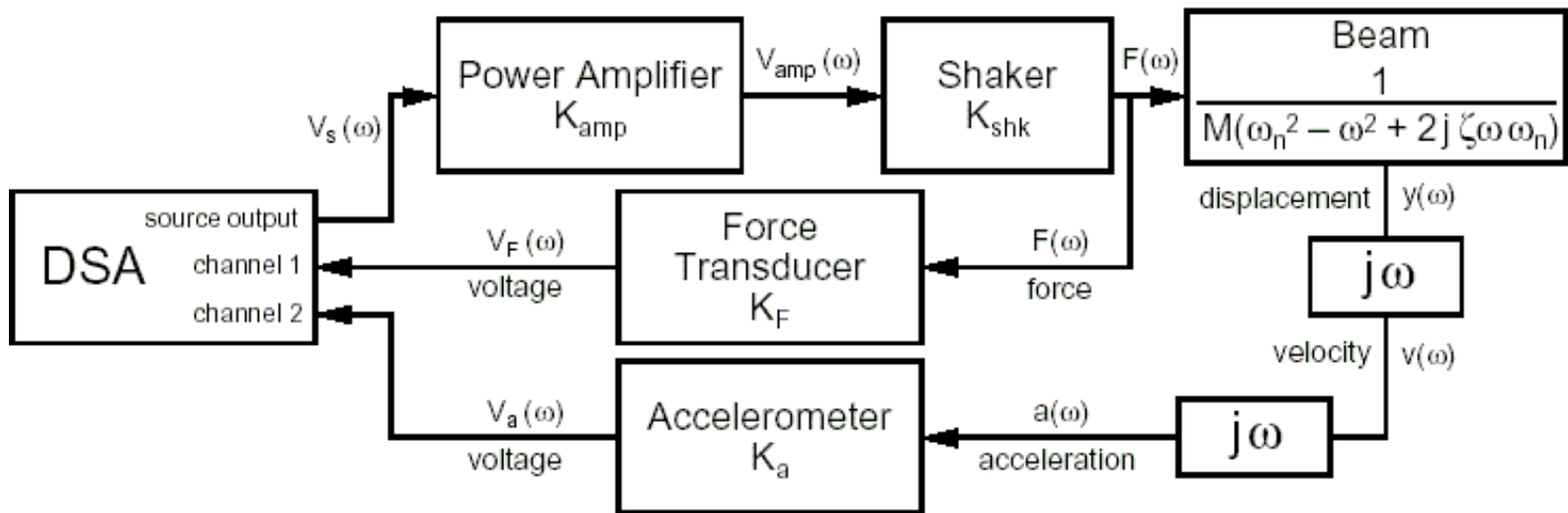


Figure C.2 Block diagram of setup used to study beam vibration characteristics.

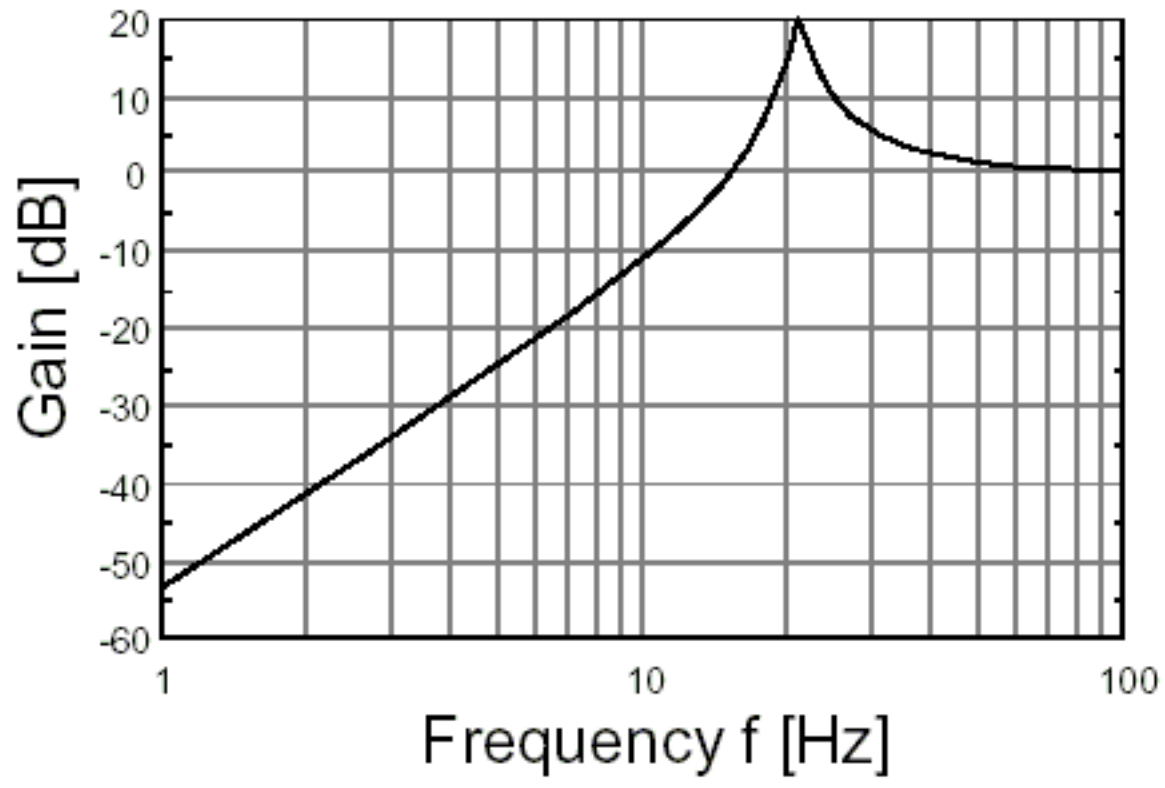


Figure C.3 Frequency Response Plot for Beam-mass System.