

# ME 460 Lab 1

## Dynamic Simulation using Simulink and MATLAB

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### 1: Introduction

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In this lab you will learn how to use Simulink to model and simulate dynamical systems. Simulink is integrated with MATLAB and uses a block diagram environment to represent dynamic systems. You can think of Simulink as a graphical programming tool. Instead of writing large amounts of code, you can simply drag-and-drop predefined blocks into the model window and connect the inputs and outputs to create a simulation.

Timing, numerical methods, and other settings can be set up in the system configuration before running the simulation in Simulink. Progress of the simulation can be monitored while running, and the final results can be exported to MATLAB for post-processing when complete.

### 2: System Simulation Basics

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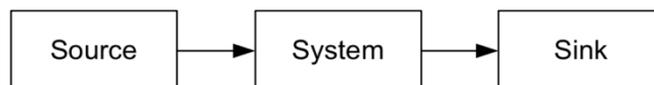
#### 2.1: Required Steps to Simulate a System

1. Determine the governing equation(s) of the system.
2. Draw out the block diagram representing the system dynamics on paper.
3. Create the corresponding block diagram in Simulink.
4. Add additional source and sink blocks to manage the inputs and outputs.
5. Setup the simulation parameters and run the simulation.
6. Display and analyze the results.

Steps 1 and 2 were completed in the Prelab assignment. The remaining steps are explained throughout the rest of the lab.

#### 2.2: Structure of a Block Diagram in Simulink

An overall block diagram in Simulink typically has the form shown in Figure 1. This figure only represents the general high-level form of a block diagram. It does not include many more detailed blocks within the system and different options for source and sink blocks.



*Figure 1: Typical Block Diagram Model in Simulink*

The system block defines all the system dynamics you want to simulate. Depending on the complexity of the system, this block may consist of one or more predefined blocks from the Simulink library.

If the system dynamics has a forced input, you will need to include a source block to supply a source which creates the desired input. The Simulink library contains several source blocks to generate a forced input such as a step input, ramp input, sine wave, or square wave.

The output signals of the system can be displayed or stored using a sink block. Again, the Simulink library provides a variety of sink blocks to choose from. For example, the *To Workspace* block stores the signal history of the specific output in the MATLAB workspace.

### 3: Exercise 1 – Spring-Mass-Damper System Comparison

This exercise will ask you to model and simulate the spring-mass-damper cart from the Prelab. With the block diagram in your answer to Question 2 of the Prelab assignment, a corresponding block diagram can be created in Simulink. Figure 2 shows what the block diagram will look like in Simulink.

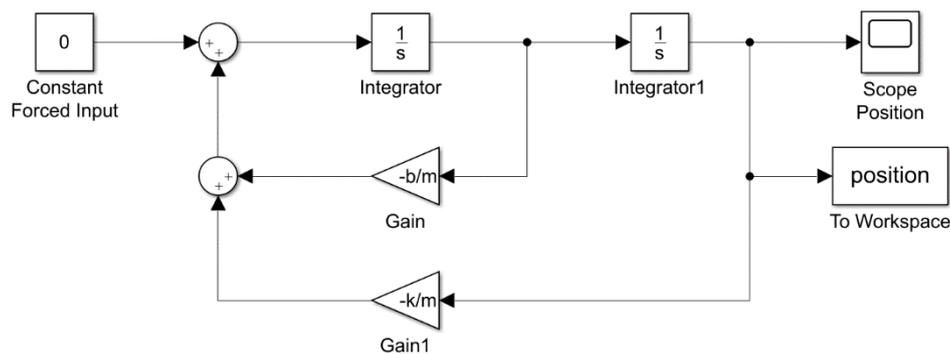


Figure 2: Simulink Block Diagram of Spring-Mass-Damper

#### 3.1: Create New Simulink Model

First run MATLAB, then start Simulink by typing `simulink` in the MATLAB command window.

```
>> simulink
```

This can also be done in MATLAB's home toolbar by clicking *New*, then *Simulink model*. The Simulink start page will open where you can select from a variety of model templates. To begin from scratch, simply click the blank model.

#### 3.2: Add Blocks from Libraries

There are two ways to add blocks from the built-in libraries:

1. Open the library browser , find the block in the corresponding library, and drag-and-drop the block into your model window.
2. Click in the white space of the model window to invoke a search bar, search for the block, and hit enter.

The second method is much faster and more convenient when the block name is known.

Add two sum blocks, two integrator blocks, and two gain blocks to your model. These are found in the *Simulink/Math Operations* and *Continuous* libraries. Arrange the blocks as shown. Click on a block and use `CTRL+R` and `CTRL+SHIFT+R` to rotate clockwise and counterclockwise.

Add *Scope*, *To Workspace*, and *Constant* blocks to your model. These are found in the *Simulink/Sources* and *Sink* libraries. The *To Workspace* block takes the connect signal's history and

stores it in the MATLAB workspace when the simulation finishes. The scope block displays the connected signal while running.

### 3.3: Connecting Blocks

Connections are made at each block's inward and outward arrows. Most connections can be made by clicking and dragging arrows along the desired route. To branch off, right-click and drag from the existing connection. Complete the connections shown in Figure 2.

### 3.4: Set Block Parameters

Set the  $-b/m$  gain block value to  $-2$  by double-clicking on the block. Set the  $-k/m$  gain block to  $-40$ . The first integrator outputs velocity. Its initial condition should be zero. The second integrator outputs position which should have an initial condition of 100 mm. Set its initial condition value to 0.1. Set the constant input to zero ( $f(t) = 0$ ) to simulate an un-forced system. Change the variable name of the *To Workspace* block to *Position*.

### 3.5: Setup Model Configuration Parameters

Click the gear icon  to pull up the model configuration parameters. In the solver settings section, change the solver to be *ode45* instead of *auto*. Leave the *Type* as *Variable-step*. Click the arrow next to *Solver details* to view more settings. Change the *Max step size* to 0.005 and the *Relative tolerance* to  $1e-5$ . Click OK to apply the changes.

### 3.6: Run Simulation and Plot Results

Run the simulation by clicking the play button. You can view the signal output while running by clicking the scope block and opening the scope's plot window. The signal output is also available in the MATLAB workspace via the to workspace block. Go to the MATLAB command prompt and type:

```
>> plot(position.time, position.data)
```

### 3.7: Save the Block Diagram

Save your model as a .slx file in your bench's directory, 'C:\ME460\_ABx\Lab1'.

Before proceeding, you should become familiar with other blocks available in the libraries. Most blocks used in this lab can be found in *Simulink/Sources*, *Sinks*, *Commonly Used Blocks*, and *Math Operations*. Block functionalities are mostly obvious by name, but to understand its use, double-click the block and then click help for a detailed explanation.

### 3.8: Inverse Laplace Transform

Compare the Simulink simulation with MATLAB's inverse Laplace transform. Take the inverse Laplace transform of  $X(s)$  using MATLAB's `impulse()` function. Here,  $X(s)$  is the Laplace transform of position of the mass-spring-damper system. The `impulse()` function computes the impulse response in the time domain of the given transfer function. Since the Laplace transform of  $\delta(t)$  is 1, this function simply computes the Inverse Laplace Transform  $x(t)$  of the given transfer function  $X(s)$ .

Write the following code in a MATLAB script file and run it multiple times with varying damping constant.

```
m = 1; % mass
k = 40; % spring constant
```

```

b = 2; % damping constant
x0 = 0.1; % initial position
v0 = 0; % initial velocity
T = linspace(0,10,200); % time vector
num = [0,m*x0,b*x0+m*v0]; % TF numerator
den = [m, b, k]; % TF denominator
x = impulse(num,den,T); % inverse laplace
plot(T,x) % plot results

```

Compare the Simulink plots to the plots generated with MATLAB's `impulse()` function.

## 4: Exercise 2 – Linear Approximation Comparison

Build a Simulink model corresponding to the block diagram from Question 4 of the Prelab. Note the equations of motion have a nonlinearity. Use the *Trigonometric Function* block from the *Math Operations* library.

This block diagram is very similar to Exercise 1. The previous gains  $-k/m$  and  $-b/m$  are now  $-g/l$  and  $-b/ml^2$ . Additionally, there is a  $\sin(\theta)$  block inserted just before the  $-g/l$  gain. This is because both systems are second order (they contain two integrators). Make the necessary modifications to Exercise 1's .slx file and save it as a new Simulink model. Run the simulation and plot your results.

Now consider a linear approximation of  $\theta$  for  $\sin \theta$ . You can comment through a block by clicking on the block and using `CTRL+SHIFT+Y`. Modify your Simulink model accordingly.

Run the linearized simulation with different initial conditions and compare it to the original nonlinear simulation. You can select all blocks, copy, and paste them to create a second simulation loop in the same Simulink model. Try plotting both linearized and nonlinear results in the same plot.

## 5: Exercise 3 – Hydraulic Motor System Comparison

Now you will develop nonlinear and linearized Simulink models of a hydraulic motor system similar to what is on the hydraulic bench. This system is shown in Figure 3 and the equations of motion will be described below.

From the setup in Figure 3, there are two states in the system:

1. The motor speed  $\omega$ .
2. The pressure downstream of the valve  $P_d$ .

The two inputs are the valve command  $u_v$ , and the pressure upstream of the valve  $u_{p_u}$ . The relationship between torque applied  $T$  to the hydraulic motor and the motor's speed  $\omega$  can be captured well with a linear first order model; see Eq. (1).

$$J\dot{\omega} = T - b\omega \quad (1)$$

$J$  is the motor inertia and  $b$  is the friction coefficient. The speed of the hydraulic motor  $\omega$  and the flow rate through the motor  $q_m$  are related by the motor displacement  $D$ ; see Eq. (2).

$$q_m = D\omega \quad (2)$$

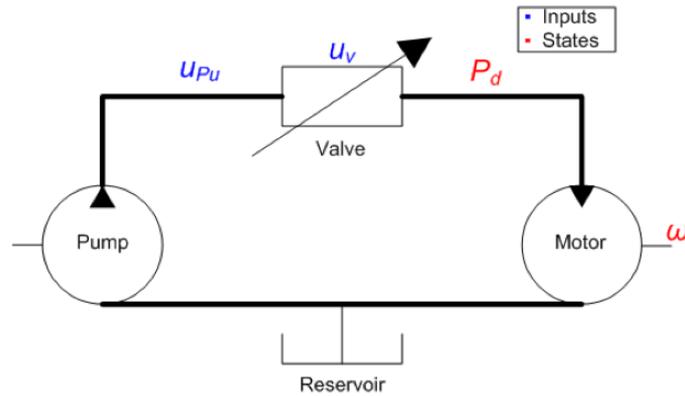


Figure 3: Hydraulic Motor System with Valve Control

Several assumptions have been made such as ignoring the leakage and static friction in the hydraulic motor. To consider leakage, a more complex relationship between flow rate and motor speed is required. These effects play a minor role in the physical system's behavior and are not necessary for deriving an adequate model of the hydraulic motor system. It is important to acknowledge what assumptions you are making when modeling a physical system.

The torque applied to the motor  $T$  is a function of the pressure drop across the motor (downstream pressure  $P_d$ ). The downstream pressure  $P_d$  has a dynamic relationship with the valve flow rate  $q_v$  and the motor flow rate  $q_m$  as shown in Equations (3) & (4).

$$\dot{P}_d = \frac{\beta}{V}(q_v - q_m) \quad (3)$$

$$T = DP_d \quad (4)$$

$V$  is the volume of oil inside the hose connection the valve to the motor and  $\beta$  is the bulk modulus of the oil (a measure of oil's resistance to compression). When the flow through the valve and the motor remain equal, the downstream pressure remains constant.

The last component of the system is the valve. The valve is actuator with a control signal  $u_v$  to vary the downstream pressure (and motor speed) for a given upstream pressure  $u_{Pu}$ . Equation (5) relates these values to the flow rate through the valve  $q_v$ .

$$q_v = ku_v \sqrt{u_{Pu} - P_d} \quad (5)$$

$k$  is the coefficient relating the valve control signal to valve opening. This equation is known as the orifice equation and can be derived from Bernoulli's equation<sup>1</sup>.

Combine Equations (1) - (5) into a system of two nonlinear dynamic equations. Linearize the system of equations about the operating point:  $u_{v0}$ ,  $u_{Pu0}$ ,  $P_{d0}$ ,  $\omega_0$ . Using the parameters from Table 1, build a Simulink model of both nonlinear and linearized versions of the hydraulic motor system. Use the same

<sup>1</sup> White, F.M., Fluid Mechanics, 5<sup>th</sup> ed. McGraw-Hill, Boston: 2013.

configuration parameters as the previous exercises. Simulate both versions with an upstream pressure of 2.75 kPa and a constant valve control signal of 2.5 V.

Table 1: Hydraulic Motor System Parameters

Bulk Modulus	$\beta$	240 MPa
Hose Volume	$V$	30 cm <sup>3</sup>
Displacement	$D$	4 cm <sup>3</sup> /rad
Friction Coefficient	$b$	1 Nms
Motor Inertia	$J$	0.5 kgm <sup>2</sup>
Valve Coefficient	$k$	2
Control Set-Point	$u_{v0}$	2.5 V
Upstream Pressure	$u_{P_u}$	2.75 MPa
Initial Downstream Pressure	$P_{d0}$	0.5 MPa
Initial Speed	$\omega_0$	0 rad/s

## 6: Report Questions

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Create a lab report addressing all the following:

1. Compare the plots from Simulink's simulation and from MATLAB's impulse function in Exercise 1. What are the differences? Should they be similar or not?
2. In Exercise 2 (Section 4), at what initial condition does the linearized model poorly approximate the original nonlinear model?
3. How are the observations you made in Question 2 (above) relevant to controller design?
4. In Exercise 3, how do the nonlinear and linearized models compare? How do they compare when changing the upstream pressure?
5. Save your plots and simulation files to your bench's directory 'C:\ME460\_ABx\Lab1'.
6. Include the resulting plots from each exercise in your lab report:
  - a. Exercise 1: Simulink plot and MATLAB plot
  - b. Exercise 2: Output signal of linearized and nonlinear systems
  - c. Exercise 3: One plot with both nonlinear and linearized pressure outputs and one plot with both nonlinear and linearized speed outputs
7. Include any additional screenshots or plots you find relevant.