Lab 9: Lead-Lag Compensator Design

Introduction

In the previous lab, we performed system identification of the XY stage. Since we have a plant model now, we can apply a control design technique to satisfy some design requirements. However, we expect errors in our system identification. In fact, we neglected static friction in the model structure used for system identification, and we did not consider other dynamics such as time delay in communication. To cope with uncertainties in our model, we can design for stability margins (e.g. gain margin and phase margin).

Our controller design uses a lead-lag compensator. A lead compensator is commonly used for improving stability margins. Lag compensators are used to improve the steady state performance. The lead compensator achieves the desired results through the merits of its phase-lead contribution. The lag compensator accomplishes its result through the merits of its attenuation property at high frequencies.

In this laboratory exercise, a lead compensator will be implemented on the XY stage.

Experimental Objectives

Completion of the laboratory exercise will have required you to:

- Design a lead compensator:

$$C(s) = K_c\left(\frac{Ts+1}{\alpha Ts+1}\right)$$

- Implement the designed compensator to control position of the XY stage. Recall the form of the plant model for both the X and Y axis:

$$G(s) = \frac{K_{sys}}{s(\tau_{sys}s+1)}$$

  where K_{sys} and τ_{sys} values where found for each axis as identified in Lab 6.

Experimental Procedure

Choose the design requirement(s)

1. Since the plant of each axis is a type 1 system, it has zero steady state error in position for the closed loop system. Therefore, we need to set the steady state velocity error requirement, $velocity\ error\ in\ steady\ state = 1/K_v$. We choose our $K_v = 100$. We choose a large value since we expect friction to reduce our ability to track this velocity.
2. For nonlinear systems, especially with poor models, a good conservative guess for the phase margin is 60^o. Recall that the relation between phase margin and damping ratio is approximated by $\zeta = \frac{\phi_m}{100}$.

3. Design for the Gain Margin to be 20 dB.

X – AXIS: Design Lead Compensator by hand

1. Find the value of K_c needed to meet the K_v requirement. Using the method described in Question 1 of the prelab.
   K_c = ___________
2. Create a transfer function in MATLAB for your model of the X-axis.
   (e.g. $sys = \frac{400}{s(0.05s+1)}$).
3. Find the phase margin and the damping ratio of your system sys combined with the controller gain K_c using the "margin" command in MATLAB. Comment on how you would expect the system to perform (steady state error and transient response) if the design stopped with this step.
   $\phi_m =$__________ $\zeta = \frac{\phi_m}{100} =$ ___________________
4. Calculate the desired amount of phase lead to add:
   $\phi_{cm} = \phi_d - \phi_m + 10^o =$_______________
5. Calculate the attenuation factor
   $\alpha = \frac{1-\sin\,\phi_{cm}}{1+\sin\,\phi_{cm}} =$_____________
6. Calculate the location of the maximum phase contribution, ω_m using your solution to Question 2 in the prelab. You may find the *roots* function in MATLAB useful.
   ω_m=______________
7. Calculate the corner time constant
   $T = \frac{1}{\omega_m\sqrt{\alpha}} =$______________
8. With this information, create the transfer function for C(s) in MATLAB.
9. Show the following plots to your TA:
   a. Plot the Bode, Nyquist and Root-Locus plots for your open loop system, C(s)G(s). Mark the relevant stability margins on these graphs by choosing *Characteristics -> All Stability Margins*.
   b. Plot the closed loop system performance using the "feedback" and "step" function in MATLAB.

Y - AXIS: Design of a Lead Compensator Using SISOTOOL

For the Y-axis you will repeat the above procedure with MATLAB's SISOTOOL.

Load the identified model of your Y-axis into MATLAB and start SISOTOOL by typing "sisotool (sys)" in the command window. Modify the controller *C* to add a lead compensator. Manually set the gain K_c for controller *C* according the method from prelab question 1. Adjust the pole and zero of your lead compensator until it satisfies the design requirements.

Note:

1. As in the lead design for the X axis, we place the location of maximum phase contribution ω_m at the new gain crossover frequency. Thus, the resulting phase margin should be located between your lead compensator's pole and zero. Keep this in mind when making adjustments in sisotool.
2. Poles at very high frequencies might not be effective since the controller is implemented to the Simulink block which runs at some finite sampling frequency (2 kHz). So place the pole of your controller where it can be implemented effectively.
3. In real system, there is time delay and other high frequency dynamics ignored in our modelling. These additional dynamics add more phase decrease in high frequency. So you might need more phase margin in your design with the nominal model, to cope with these types of uncertainties.
4. Show the following plots to your TA:
    a. Plot the Bode, Nyquist and Root-Locus plots for your open loop system, C(s)G(s).
    b. Plot the closed loop system performance.
5. Export your controller from *SISOTOOL* to *Matlab workspace*.

Implementation of the designed controller in XY stage position control
Use LTI system block to implement the controller in the Simulink model, xy_stage_start.slx in N:\Hydrauliclab\ME460. Use the following position reference signals:

1. Square wave with 5 mm amplitude at 0.25 Hz (each axis individually)
2. Circle trajectory used in Lab 7 with PID control

Try to increase the gain of controller for faster response and smaller steady state error. Check what gain gives undesirable behavior (Unstable response, jittering, etc) and compare it with the gain margin you had in controller design.

Lab Report

1. Write down your design requirement(s) and show your work in designing your controller. In addition, present the Bode plot and transfer function of the designed compensator with the plant model.

2. Compare the position tracking plot of the real XY stage with a Simulink simulation and discuss what causes the discrepancies. Does the real XY stage meet the steady-state velocity error requirement? (Make sure to attach the plot of the signals: output, reference, control.)

3. Were you able to increase the gain in the controller on the real XY stage up to the GM in your design? If you couldn't then explain why it is different from what was expected.

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