Prelaboratory Exercise 4

Objective

In this Prelab you will complete the background work required for the root locus based controller that you will design and implement during Lab 4. You will also be asked to include these results in your Lab 4 report.

Assignment

Read the background information contained in this Prelab and answer each question. Note that the questions are contained throughout the document.

Background

The theoretical background for the root locus method should have been covered in class and only a review will be presented here. For more information on the Evans' Root Locus, see Ogata 5th ed., Sections 6-1 to 6-5.

Basic Theory and Notation

Closed loop pole locations in the complex plane (s-plane) are a function of the controller's gains. Pole locations in the s-plane are important because they characterize the system's response to a step input. The root locus plot traces the motion of these poles as the compensator gains are varied from zero to infinity.

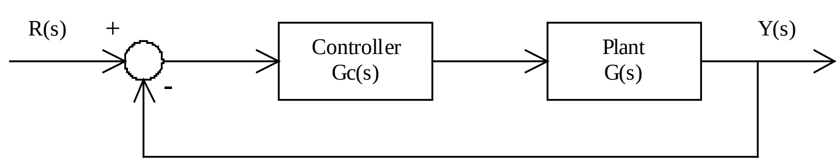

Figure 1: A block diagram of a closed loop system with unity feedback.

Question 1: What is the characteristic equation for the closed loop system shown in Figure 1?

The characteristic equation can be rearranged as shown in Equation 1

$$1 + K_c \frac{num(s)}{den(s)} = 0 \qquad (1)$$

where K_c is the control gain whose effect on the poles of the closed loop system we wish to observe.

Pole Location vs. System Response

The location of the poles has a direct relationship to the response of a system. Franklin (p. 121) has an excellent picture explaining the qualitative effect of pole location on the system's step response. It is reproduced here as Figure 2.

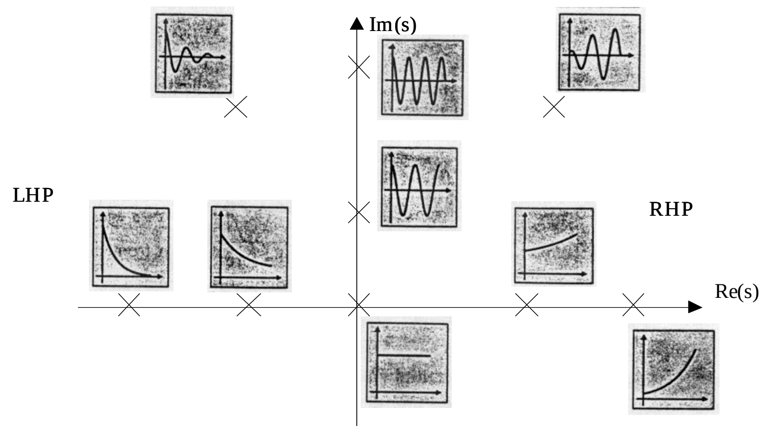

Figure 2: First order impulse response v. pole location**.**

For a system with only one pole, we see that a pole at the origin has a constant step response. By pushing this pole into the LHP (left half-plane) along the real axis, we note that the response of the system becomes faster, i.e., the time constant, τ, of the system decreases. Poles in the RHP (right half-plane) have exponentially increasing responses which lead to system instability. Moving the pole up the imaginary axis causes the system's response to becomes purely imaginary. The frequency of the oscillations increase as the distance to the origin increases. System response due to pole locations not on either axis is a combination of the oscillatory influence of the imaginary term and the exponential behavior of the real term.

Second Order System Characterization

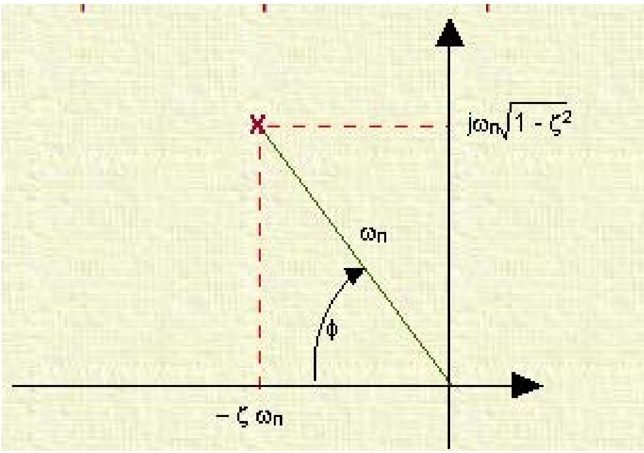

Figure 3: Pole location vs. ζ and ω_n.

Two parameters, ζ and ω_n, characterize a second order response. Their values are related to the poles of a system as shown in Figure 3.

Desired second order system responses are rarely specified in terms of ζ and ω_n. Instead, the requirements will specified by such characteristics as overshoot M_p, and rise time t_r (Ogata 5th ed. p. 169-170). The relationships between all of these values appear in Ogata 5th ed. (p. 171-176). Note that these relationships hold only for second order systems.

Overview of Design Process

Design of a compensator using the root locus design method typically involves the following steps.

- Obtain a plant model.
- Translate design requirements into ζ and/or ω_n information. System characteristics such as time constant (for first order systems) and overshoot and rise time (for second order systems) are design requirements.
- Translate ζ and/or ω_n information into closed loop pole locations.
- Draw the root locus plot of the compensated system for a range of gains K_c.
- Select a gain from the root locus plot which corresponds to the desired pole locations and use it in the compensator.
- Iterate if the system characteristics are not met due to unmodeled dynamics.

You will complete the second and third steps of the compensator design process in this Prelab. The open loop hydraulic motor system that utilized the BD90 valve, as identified in Lab 2 at 300 psi, should have a time constant of about 50 ms and a steady state gain of about 0.60. For Exercise 1 in Lab 4, we would like the time constant to be 25 ms. In Lab 4 we will close the loop and implement a proportional only controller because there is no requirement on the steady state error or overshoot.

Question 2:

(a.) What is the open loop transfer function for the motor system based on the time constant and gain specified above?

(b.) Where is the open loop pole for the motor system? How are the time constant and the open loop pole related?

(c.) What closed loop transfer function pole do we desire so that we get a time constant of 25 ms? Use the relationship between τ and s that you found in part (b).

For Exercise 2 in Lab 4, we will design a second controller for a different set of design requirements. This will be a proportional-integral (PI) controller. The new requirements are that the system response have an overshoot, M_p, of exactly 1.5% and a peak time, t_p, of approximately 0.21 seconds.

Question 3:

(a.) Translate the overshoot and peak time into ζ and ω_n values. Calculate the desired complex closed loop poles.

(b.) What is the open loop transfer function for the motor + control system? Where are the open loop poles for this system?