# Prelaboratory Exercise 6

**Objective**

In this lab you will identify the two transfer functions of the XY stage located in the hydraulics lab. The first transfer function will be of the belt driven Y stage and the second transfer function will be of the lead-screw driven X stage.

**Background**

For the last three labs of the semester, you will be working with the XY stages found in the hydraulics lab. By lab eight you will be commanding this XY stage to follow a desired trajectory. Each stage is driven by a brushless DC motor. For the Y stage, the motor is connected first to a gear reduction and then to a belt to drive the Y motion. For the X stage, its motor is also geared and then attached to a lead-screw to create the X motion.

In order to simulate this system in the next two labs, you will identify the transfer functions that relate the X motion's command voltage (*proportional to force*) to millimeters traveled in the X direction and the Y motion's command voltage (*proportional to force*) to millimeters traveled in the Y direction. For this identification you will be ignoring the static friction and the coupling forces between the two axes and just identify one direction's transfer function at a time. So the identification will not be perfect, but close enough to run simulations that approximate the system. The open loop transfer function that you will identify for X axis system has the form:

$$\frac{X_{mm}}{V_{cmd}} = \frac{K_{sys}}{s(\tau_{sys}s+1)}.$$

The Y axis will have the same transfer function but with a different $K_{sys}$ and $\tau_{sys}$. Thinking about this transfer function, if you give an open-loop step V command to the X stage the stage will continue to move in the X direction until it meets the end of its travel and stops. That open loop response is not too helpful in identifying the system. Also if we apply a sinusoidal input to the system it may oscillate back and forth a few times but then drift into the limits of the travel of the X stage hindering the identification of the open-loop system.

The solution to the identification of this type of system, which has position output instead of the velocity output transfer functions, is to identify a closed-loop system. The closed-loop system will keep the stage inside the limits of its travel. Then once the closed-loop system is identified the parameters of the open-loop transfer function can be solved.

The identification steps to identify this system is to first control the X position (or Y position) with a simple proportional controller. Produce a step response of this closed-loop system and first identify the parameters, $\xi$ $and$ $\omega_n$, of this closed-loop response. Then since the "Kp" proportional gain you can find the closed-loop transfer function with the parameters Kp, $K_{sys}$ and $\tau_{sys}$. Finally using the values found for $\xi$ $and$ $\omega_n$ solve for $K_{sys}$ and $\tau_{sys}$

**Question 1:** Find the unity feedback closed loop transfer function of a proportional controller, with gain Kp, controlling the transfer function $\frac{K_{sys}}{s(\tau_{sys}s+1)}$. Show that it is in the standard second order system transfer function form $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$.

**Question 2:** Using the results you found in question 1 and given that you know Kp, solve for $K_{sys}$ and $\tau_{sys}$ in terms of Kp, $\xi$ *and* $\omega_n$.

**Question 3:** Practice identifying $\xi$ *and* $\omega_n$ from a step response plot. Show a plot and all your calculations. In Matlab set $\xi =$ 0.2 and $\omega_n$ to 20. Then create the transfer function $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$. Use "step" to produce a step response of this transfer function. The frequency of the decaying oscillation is called $\omega_d$ which is equal to $\omega_n\sqrt{1-\xi^2}$. Measure and verify $\omega_d$. To find $\xi$ you will use a method called "logarithmic decrement" which uses the fact that the peaks of the step response are exponentially decaying. Your TA will explain this more in lab, but to find $\xi$ measure the amplitude value (distance from steady state value) of two adjacent amplitudes with amp1 being the larger amplitude and solve $\delta = \ln(\frac{amp1}{amp2})$ and $\xi = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$. Also see https://www.andrew.cmu.edu/course/24-352/Handouts/logdecrement.pdf for an explanation of these equations.