Prelaboratory Exercise 8

Objective

In this Prelab you will start the design of a lead compensator based on a given model and specific design requirements. You will implement this lead compensator during Lab 8.

References

Lecture notes: Lead Compensator & Lag Compensator

Basic Theory and Notation

A lead compensator has the form:

\[ C(s) = K_c \left( \frac{T s + 1}{\alpha T s + 1} \right) \]  

with \(0 < \alpha < 1\). You can plot the Bode diagram (shown in Figure 1) to see how it contributes to the gain of the system.

![Bode diagram of lead compensator](image)

Figure 1-Bode diagram of lead compensator
In this lab, lead compensators will be the controllers and our plants will be the axes of the XY stage. The control for a single stage is shown in Figure 2.

Figure 2-Closed Loop System

**Overview of Design Process**

Design of a lead compensator for time domain specifications typically involves the following steps:

- Obtain a plant model and design requirements (e.g. steady state error, phase margin, and gain margin).
- Determine the dc gain of the controller based on the steady state error requirement.
- Draw the Bode plot of the open loop system with the dc gain selected, then find what phase lead $\phi_{cm}$ needs to be added to satisfy phase margin requirement. Add a lead compensator according to the desired phase lead, $\phi_{cm}$ and the maximum phase contribution frequency, $\omega_m$, using the following equations.

\[
\alpha = \frac{1 - \sin \phi_{cm}}{1 + \sin \phi_{cm}}, \\
20 \log_{10} |C(j \omega_m)G(j \omega_m)| = 0, \\
T = \frac{1}{\sqrt{\alpha} \omega_m}
\]

**Design of a Lead Compensator**

Consider a system of the form:

\[
G(s) = K_{pt} \frac{\omega_{pt}}{s(s + \omega_{pt})}
\]  

(2)

where $K_{pt}$ and $\omega_{pt}$ are the plant’s gain and corner frequency. Use controller $C(s)$ with $G(s)$.

**Question 1:** As part of the design of a lead controller, you need to calculate the static error constants of the closed loop system. In this case, suppose we have a static velocity error constant specification $K_v$. Solve for $K_c$ in terms of $K_v$ and the coefficients of $C(s)$ and $G(s)$ using the final value theorem: $K_v = \lim_{s \to 0} sC(s)G(s)$. 

Question 2: Since we want $\omega_m$ to be our new gain crossover frequency we need, 

$$20 \log_{10} |C(j\omega_m)G(j\omega_m)| = 0 \text{ when } T = \frac{1}{\sqrt{\alpha \omega_m}} \text{ and } 20 \log_{10} |C(j\omega_m)| = 20 \log_{10} K_c + 10 \log_{10} \frac{1}{\alpha} \quad \text{Find the equation expressing the gain of the plant in decibels. Use these equations to solve for } \omega_m.$$ 

Recall: 

$$20 \log_{10} \left| K_1 \frac{2}{j(\omega + 2)} \right| = 20 \log_{10} |K_1| + 20 \log_{10} |2| - 20 \log_{10} |j\omega + 2|$$

$$= 20 \log_{10} K_1 + 20 \log_{10} 2 - 20 \log_{10} \sqrt{\omega^2 + 2^2}$$
Question 3: Sketch Bode plots for both $G(s)$ and the combination $C(s)G(s)$. You do not have to plug in values for any variables, simply label your axes with expressions of variables as done in Figure 1.

(Assume $\omega_m$ is located at $\omega_{pl}$ for this question. During the lab, you will calculate $\omega_m$ according to your answer in Question 2.)