Prelaboratory Exercise 8

Objective

In this Prelab you design a lead compensator based on a model provided to you as well as specific design requirements. You will implement this lead compensator during Lab 8.

References

Lecture notes: Lecture 21, Lead-Lag Example 1.

Basic Theory and Notation

A lead compensator has the form:

$$C(s) = K_C\left(\frac{Ts+1}{\alpha Ts+1}\right) \tag{1}$$

with $0 < \alpha < 1$. You can plot the Bode diagram (shown in Figure 1) to see how it contributes to the gain of the system.

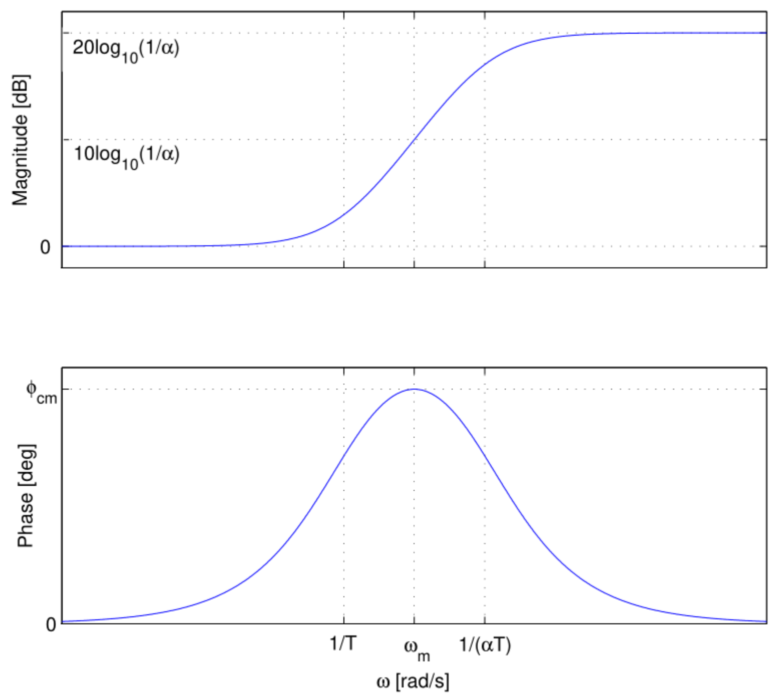

Figure 1-Bode diagram of lead compensator

In this lab, lead compensators will be the controllers and our plants will be the axes of the x-y stage. The control for a single stage as shown in Figure 2.

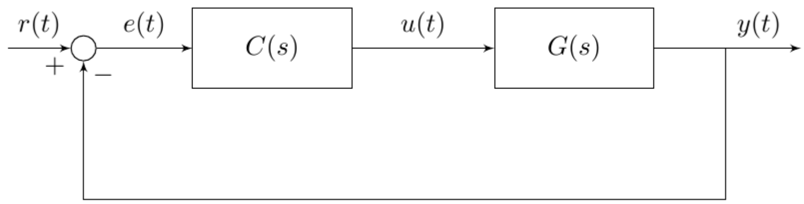

Figure 2-Closed Loop System

Overview of Design Process

Design of a lead compensator for time domain specifications typically involves the following steps:

- Obtain a plant model and design requirement. (e.g. steady state error, phase margin, and gain margin)
- Determine the dc gain of controller based on the steady state error requirement.
- Draw Bode plot of the open loop system with the dc gain selected, and find what frequency needs phase lead to satisfy phase margin requirement. Add a lead compensator according to the desired phase lead, ϕ_{cm} and the frequency, ω_m, using the following equations.

$$\alpha = \frac{1 - \sin\ \phi_{cm}}{1 + \sin\ \phi_{cm}}, \qquad 20\log_{10}|C(j\omega_m)G(j\omega_m)| = 0\,, \qquad T = \frac{1}{\sqrt{\alpha}\omega_m}$$

Design of a Lead Compensator

Consider a system of the form:

$$G(s) = K_1 \frac{\omega_1}{s(s+\omega_1)} \tag{2}$$

along with the controller dynamics, $C(s)$.

Question 1: As part of the design of a lead controller, you need to calculate the static error constants of the closed loop system. In this case, suppose we have a static velocity error constant specification K_v. Solve for K_c in terms of the coefficients of C(s) and G(s) using the final value theorem: $K_v = \lim_{s\to 0} sC(s)G(s)$.

Question 2: Since we want $\boldsymbol{\omega_m}$ to be our new gain crossover frequency we need , $20\log_{10}|C(j\omega_m)G(j\omega_m)| = 0$ when $T = \frac{1}{\sqrt{\alpha}\omega_m}$ and $20\log_{10}|C(j\omega_m)| = 20\log_{10} K_c + 10\log_{10}\frac{1}{\alpha}$. **Find the equation expressing the gain of the plant in decibels. Use these equations to solve for $\boldsymbol{\omega_m}$.**

Recall: $20\log_{10}\left|K_1\frac{2}{(j\omega+2)}\right| = 20\log_{10}|K_1| + 20\log_{10}|2| - 20\log_{10}|j\omega+2|$

$$= 20\log_{10} K_1 + 20\log_{10} 2 - 20\log_{10}\sqrt{\omega^2+2^2}$$

Question 3: Sketch Bode plots for both G(s) and the combination C(s)G(s) assuming that $\boldsymbol{\omega_m}$ of C(s) at $\boldsymbol{\omega_1}$.