Suggested Reading:

Study the Matlab help pages for the following functions: sisotool, tf, zpk, pzplot and rlttool.

1. (Both partners must include this in their Prelab 8 submission for credit. Submit this project explanation on a separate piece of paper so that the rest of your prelab can be graded while I am reading through your projects.) Explain what you and your partner would like to do for your end of the semester project. Your project MUST use a MSP430 processor. It can also include the robot car, the Beaglebone board, the myRIO board and other hardware you have in your possession. We only have 8 robots in the lab, 8 Beaglebone boards and 8 myRIos, so everyone will not be able to use all of these in their projects. Write a few paragraphs explaining your idea and add sketches if that will help explain your idea. The lab has a number of different sensors and actuators you can use in your project. Give me your ideas and after reading your proposal I will help mold your project into a form that is doable before the end of the semester and with sensors that are easily purchased or already owned by the lab. You should try to have both sensing and actuation in your project.

2. In MATLAB solve the following over-determined set of linear equations for a, b, c, d using least squares.

\[
\begin{align*}
543 &= 34a + 56b + 91c + 34d \\
678 &= 129a + 52b + 12c + 9d \\
145 &= 482a + 90b + 142c + 92d \\
118 &= 592a + 143b + 284c + 311d \\
292 &= 210a + 832b + 97c - 123d \\
95 &= -342a + 89b - 123c - 382d \\
399 &= -56a - 34b + 92c + 81d
\end{align*}
\]

Type “help slash” to see how to solve an over-determined set of equations in MATLAB. Show that the answer is a = -0.4742, b = -0.0938, c = 3.4708, and d = -1.1923 by printing out the MATLAB commands and their output you used to find this answer. Note, these numbers and equations were just pulled out of the air as an exercise to familiarize you with a least-squares solution.

3. Write out the Matlab commands for creating the following transfer functions in Matlab. Use 50ms sampling time.

\[
G(z) = \frac{2z^2 - z + 4}{z^3 + 5z^2 - z + 1} \quad H(z) = \frac{(z - 0.25)}{(z + 0.5)(z - 0.7)}
\]
4. Sketch or plot in Matlab the pole-zero plots of the transfer functions in problem 3. Indicate whether each is stable and/or non-minimum phase (zeros outside the unit circle). You may use Matlab to check your answers.

5. We will model the robot car as the transfer function \( G(s) = \frac{K}{\tau s + 1} = \frac{V(s)}{U(s)} \) where the input, \( U \), is the sum of the control effort (torque) applied to the wheels and the output, \( V \), is the speed of the car. Using the zero-order hold (ZOH) to discretize this transfer function gives the discrete transfer function \( \frac{V(z)}{U(z)} = \frac{K[z^{-1} - e^{-\tau T/z}]}{z - e^{-\tau T/z}} \) where \( K \) and \( \tau \) are from the continuous transfer function and \( T \) is the sample period.

Show that this discrete transfer function results in a difference equation (which can be implemented in a microprocessor!) given by \( v(n) = c_1 v(n-1) + c_2 u(n-1) \), where \( c_1 \) and \( c_2 \) are constants. What are \( c_1 \) and \( c_2 \) equal to in terms of \( K \), \( T \) and \( \tau \)?

6. For use in our root locus design in lab, derive a single transfer function (from error to control input) of a discrete PI controller (using the Tustin rule) in terms of the controller gains \( K_p \) and \( K_i \) and the sample period \( T \).

7. Show that the discrete characteristic equation of the first-order plant and the discrete PI controller is given by the equation

\[
1 + C(z)G(z) = 1 + \frac{2K_p + K_i T}{2} \left( \frac{z + \frac{K_i T - 2K_p}{2K_p + K_i T}}{z - 1} \right) \left( \frac{c_2}{z - c_1} \right) = 1 + K_{loop} \left( \frac{c_2}{z - c_1} \right) \left( \frac{z - zero_c}{z - 1} \right)
\]

8. Express \( K_i \) (the integral gain) and \( K_p \) (the proportional gain) in terms of \( K_{loop} \) (the loop gain of the characteristic equation) and \( zero_c \) (the compensator zero).