ME 461 Prelab #8
Fall 2017
Due at the beginning of class 11/15/2017

Suggested Reading:

Study the Matlab help pages for the following functions: `sisotool`, `tf`, `zpk`, `pzplot` and `rltool`.
Lab 8 manual

1. In MATLAB solve the following over-determined set of linear equations for a, b, c, d using least squares.

```
543   =     34a       + 56b      + 91c      + 34d
678   =     129a      + 52b      + 12c      + 9d
145   =     482a      + 90b      + 142c     + 92d
118   =     592a      + 143b     + 284c     + 311d
292   =     210a      + 832b     + 97c      - 123d
95    =     -342a     + 89b      - 123c     - 382d
399   =     -56a      - 34b      + 92c      + 81d
```

   Type "help slash" to see how to solve an over-determined set of equations in MATLAB. Show that the answer is a= -0.4742, b = -0.0938, c = 3.4708, and d = -1.1923 by printing out the MATLAB commands and their output you used to find this answer. Note, these numbers and equations were just pulled out of the air as an exercise to familiarize you with a least-squares solution.

2. Write out the Matlab commands for creating the following transfer functions in Matlab. Use 5ms sampling time.

$$G(z) = \frac{2z^2 - z + 4}{z^3 + 5z^2 - z + 1} \qquad H(z) = 5\frac{(z - 0.25)}{(z + 0.5)(z - 0.7)}$$

3. Sketch or plot in Matlab the pole-zero plots of the transfer functions in problem 2. Indicate whether each is stable and/or non-minimum phase (zeros outside the unit circle). You may use Matlab to check your answers.

4. We will model the robot car as the transfer function $G(s) = \frac{K}{\tau s+1} = \frac{V(s)}{U(s)}$ where the input, U, is the sum of the control effort (torque) applied to the wheels and the output, V, is the speed of the car. Using the zero-order hold (ZOH) to discretize this transfer function gives the discrete transfer function $\frac{V(z)}{U(z)} = \frac{K\left[1 - e^{-T/\tau}\right]}{z - e^{-T/\tau}}$ where K and τ are from the continuous transfer function and T is the sample period.

Show that this discrete transfer function results in a difference equation (which can be implemented in a microprocessor!) given by $v(n) = c_1 v(n-1) + c_2 u(n-1)$, where c_1 and c_2 are constants. What are c_1 and c_2 equal to in terms of K, T and τ?

5. For use in our root locus design in lab, derive a single transfer function (from error to control input) of a discrete PI controller (using the Tustin rule) in terms of the controller gains K_p and K_i and the sample period T. Make sure that your transfer function is not the sum of two transfer functions. Find a common denominator and derive a single transfer function.

6. Show that the discrete characteristic equation of the first-order plant and the discrete PI controller is given by the equation

$$1 + C(z)G(z) = 1 + \frac{2K_p + K_I T}{2}\left(\frac{z + \frac{K_I T - 2K_p}{2K_p + K_I T}}{z - 1}\right)\left(\frac{c_2}{z - c_1}\right) = 1 + K_{loop}\left(\frac{c_2}{z - c_1}\right)\left(\frac{z - zero_c}{z - 1}\right)$$

7. Express K_i (the integral gain) and K_P (the proportional gain) in terms of K_{loop} (the loop gain of the characteristic equation) and $zero_c$ (the compensator zero).