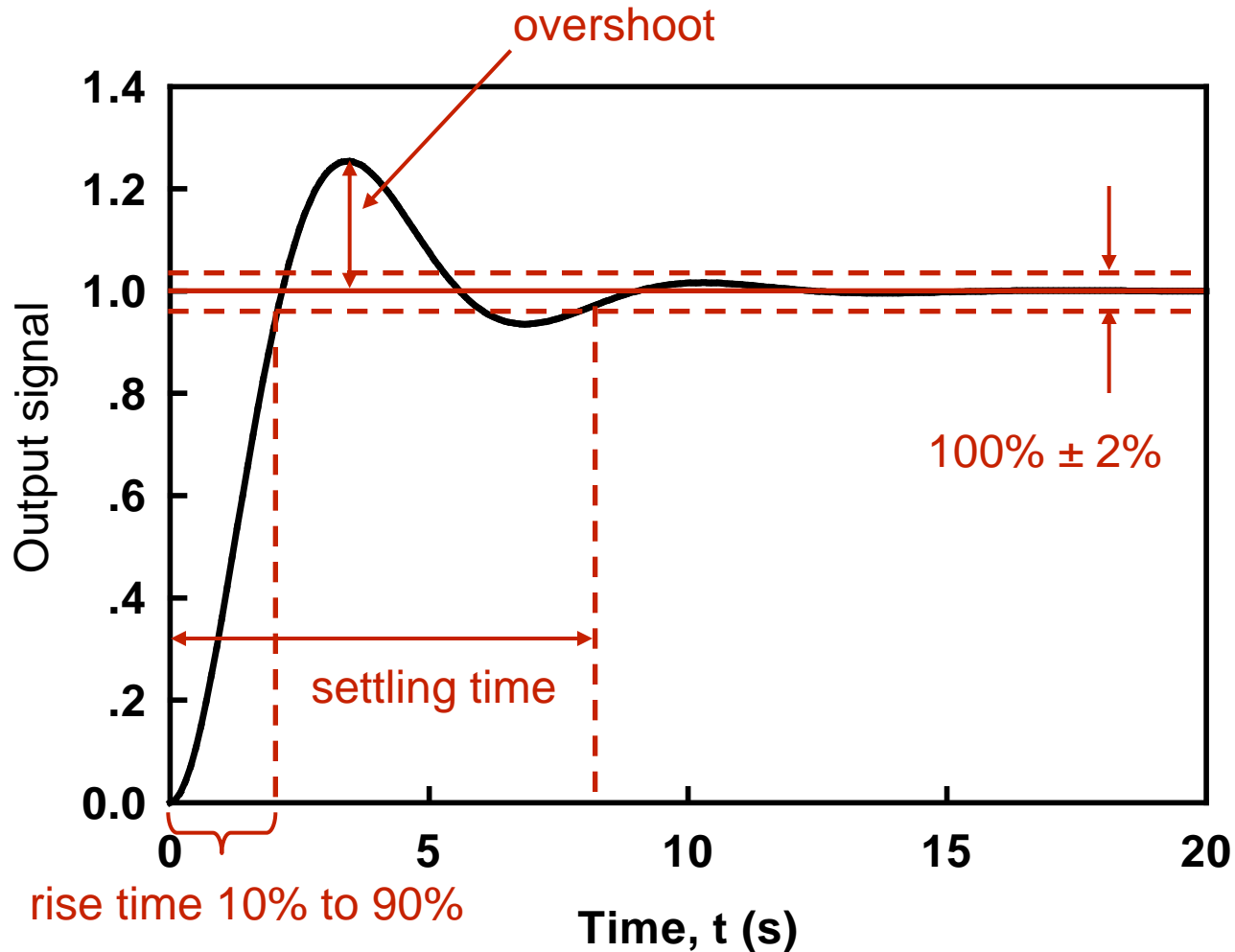
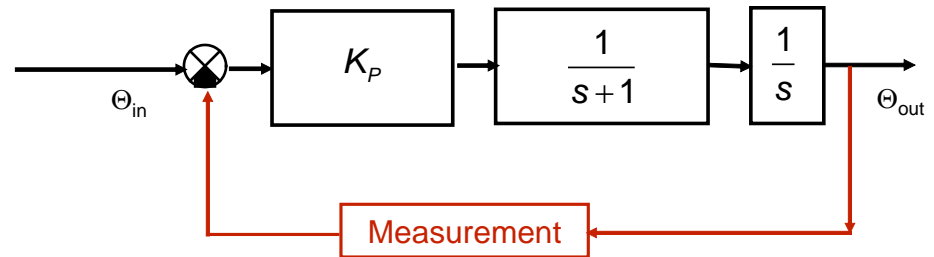


Second-Order Step Response



Matlab & Controller Design

Plant: $G(s) = \frac{1}{(s+1)s}$



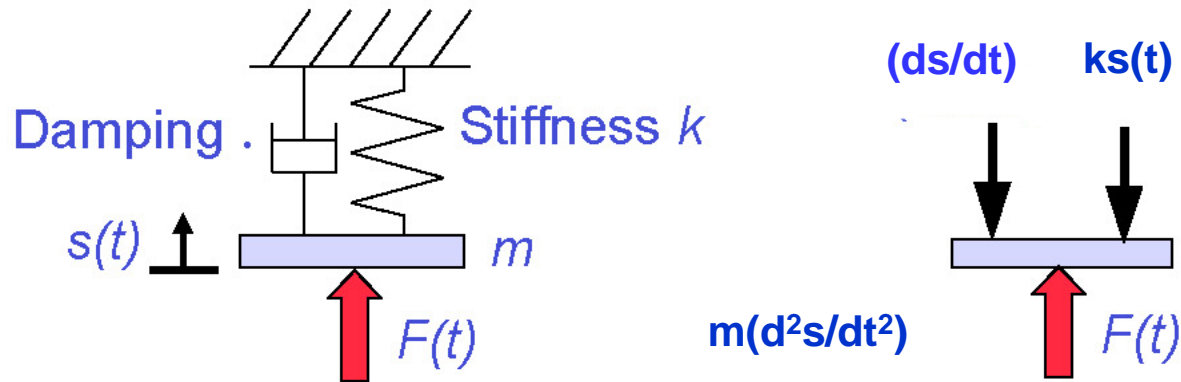
Compensator: $D(s) = K_P$

Transfer Function: $TF = \frac{K_P \left(\frac{1}{(s+1)s} \right)}{1 + K_P \left(\frac{1}{(s+1)s} \right)}$

Matlab: `sysGs = tf([1], [1 1 0]);`
`rltool(sysGs);`

Second-Order Systems

- Recall: A simple mechanical system:



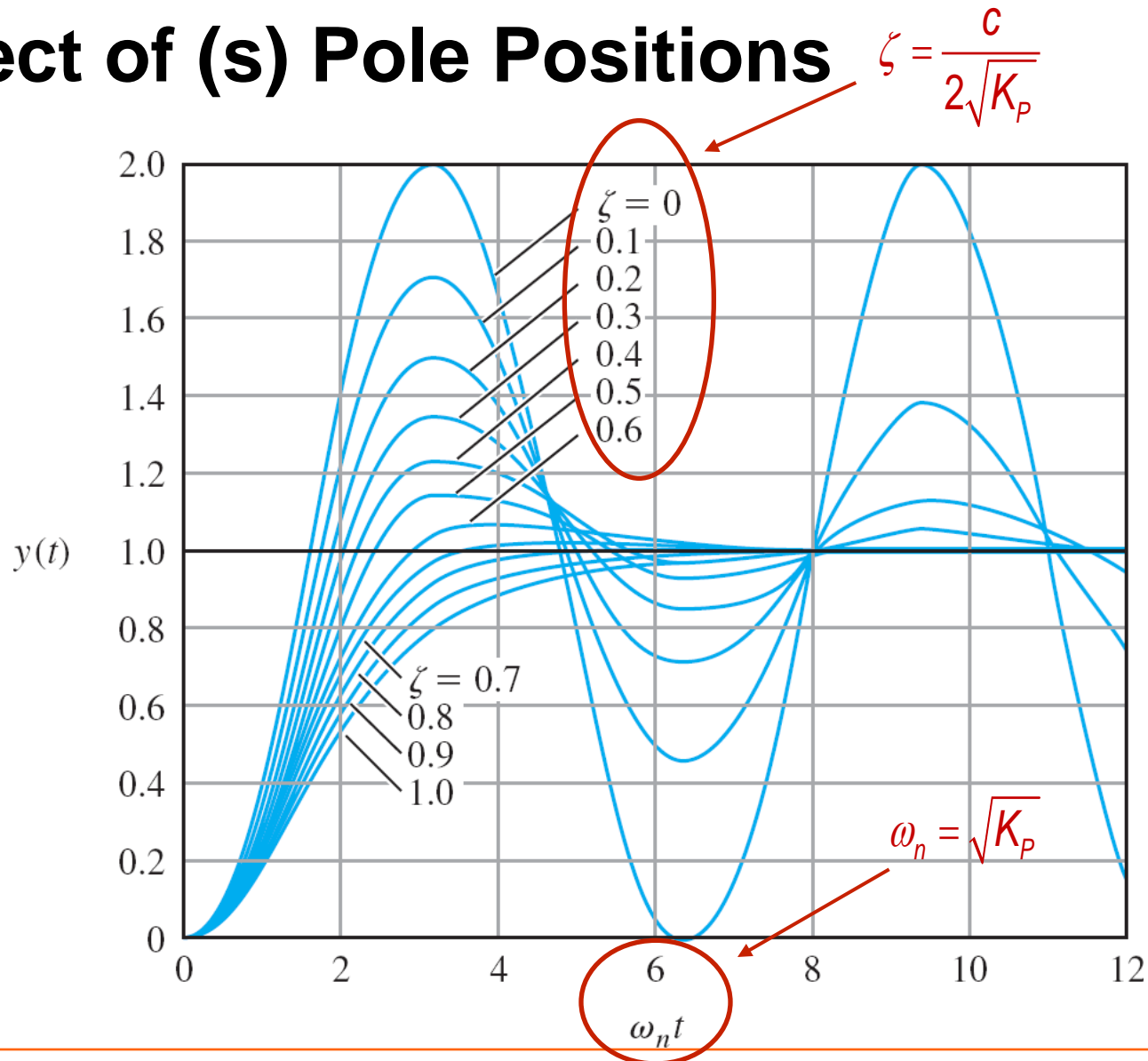
$$m \frac{d^2 s}{dt^2} + c \frac{ds}{dt} + ks = F(t)$$

$$K = \frac{1}{k} \quad \text{m/N}$$

$$\zeta = \frac{c}{2\sqrt{km}} \quad (\text{dimensionless})$$

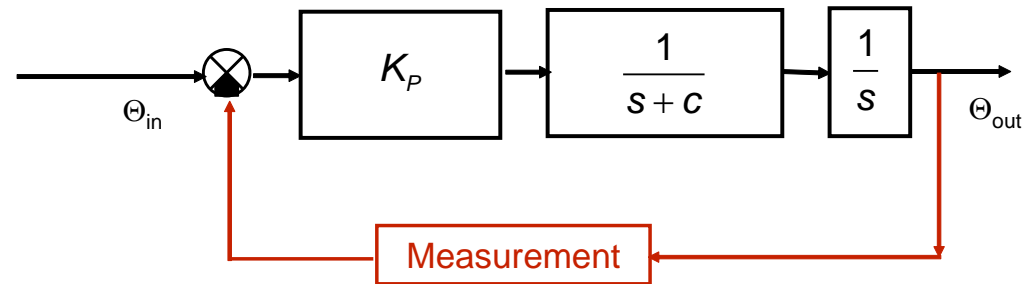
$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{rads/s}$$

Effect of (s) Pole Positions



Effect of (s) Pole Positions

- System Block Diagram



- Closed-Loop Transfer Function:

$$TF = \frac{K_P}{s^2 + sc + K_P}$$

- Corresponding Differential Equation:

$$\frac{d^2\theta_{out}}{dt^2} + c\frac{d\theta_{out}}{dt} + K_P\theta_{out} = K_P\theta_{in}$$

$$K=1 \quad \zeta = \frac{c}{2\sqrt{K_P}} \quad \omega_n = \sqrt{K_P}$$

Express the TF denominator as:

$$\begin{aligned} & (s + \sigma - j\omega_d)(s + \sigma + j\omega_d) \\ &= s^2 + 2\sigma s + (\sigma^2 + \omega_d^2) \end{aligned}$$

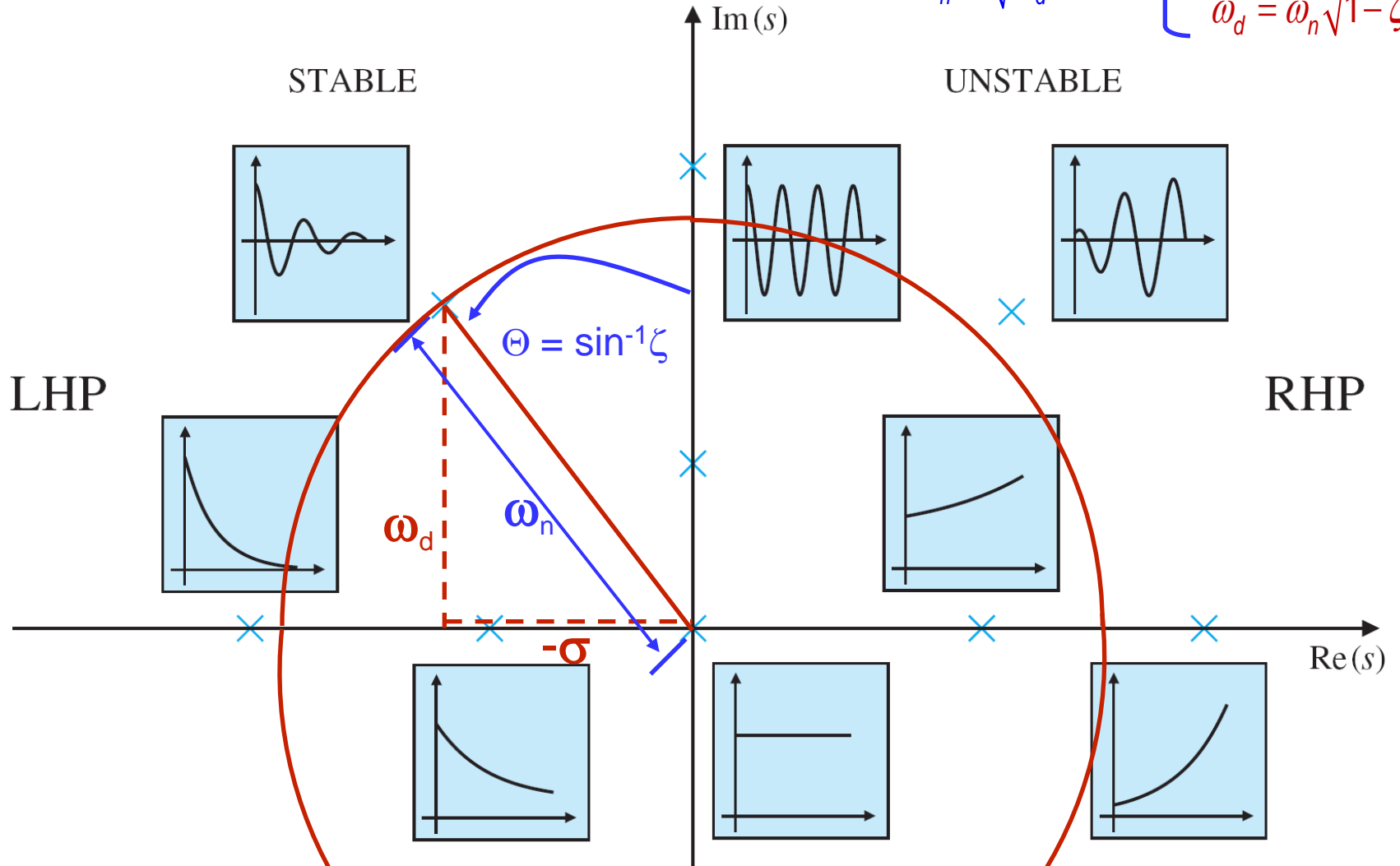
$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$s = -\sigma + j\omega_d$$

Effect of (s) Pole Positions

$$\omega_n = \sqrt{\omega_d^2 + \sigma^2} \begin{cases} \sigma = \zeta\omega_n \\ \omega_d = \omega_n\sqrt{1-\zeta^2} \end{cases}$$



Z-Transform

- Discrete analog to the continuous Laplace Transform
- Transforms linear difference equations to algebraic equations that are much easier to solve
- Enables dynamic analysis of discrete systems using familiar transform tools (root locus, Nyquist, Bode)

$$\mathcal{Z}\{f(k)\} = F(z) = \sum_{k=0}^{\infty} f(k) z^{-k}$$

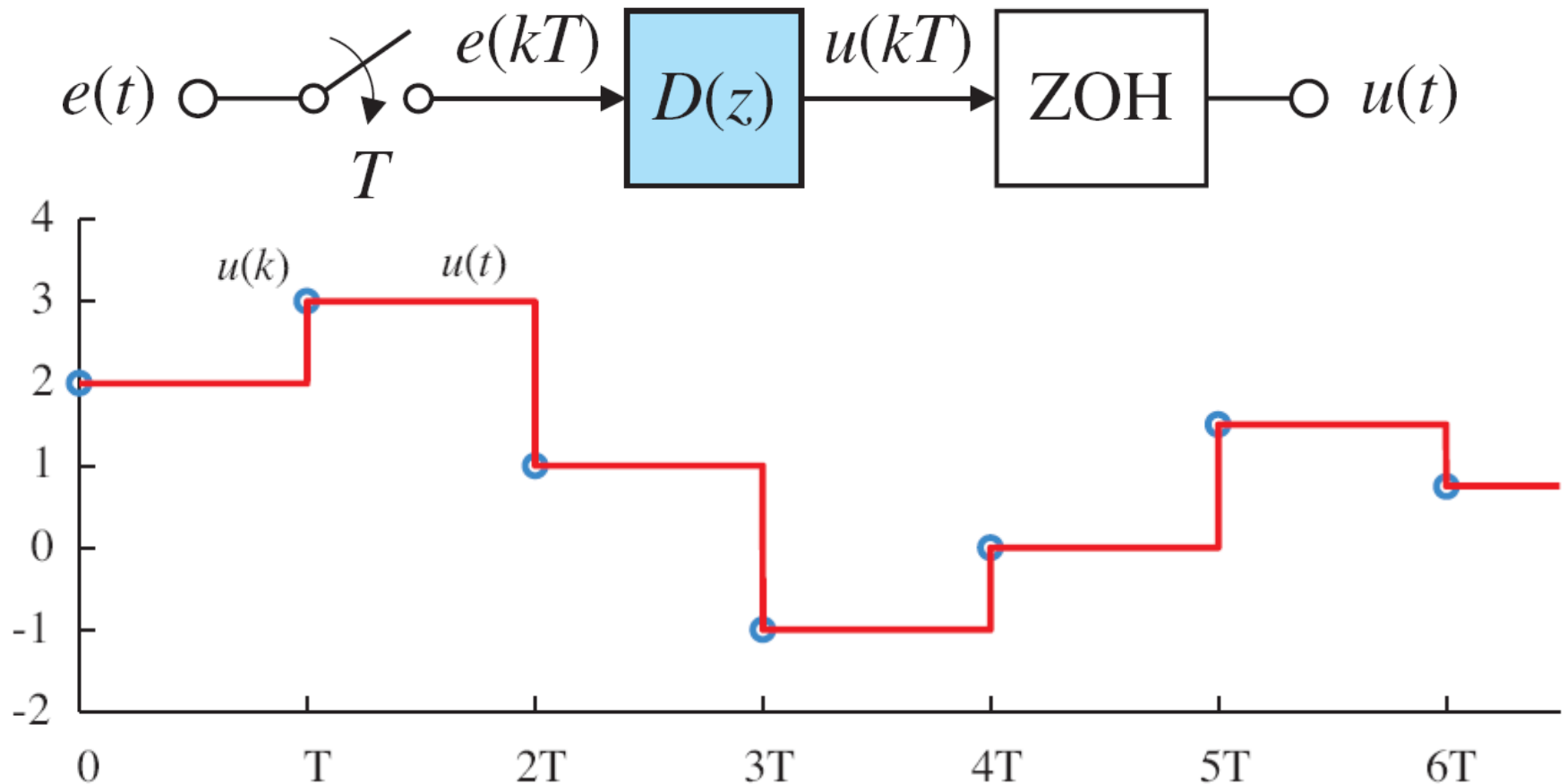
- This leads directly to:

$$\mathcal{Z}\{f(k-1)\} = z^{-1}F(z) \quad z^{-1} \text{ can be considered a shift operator}$$

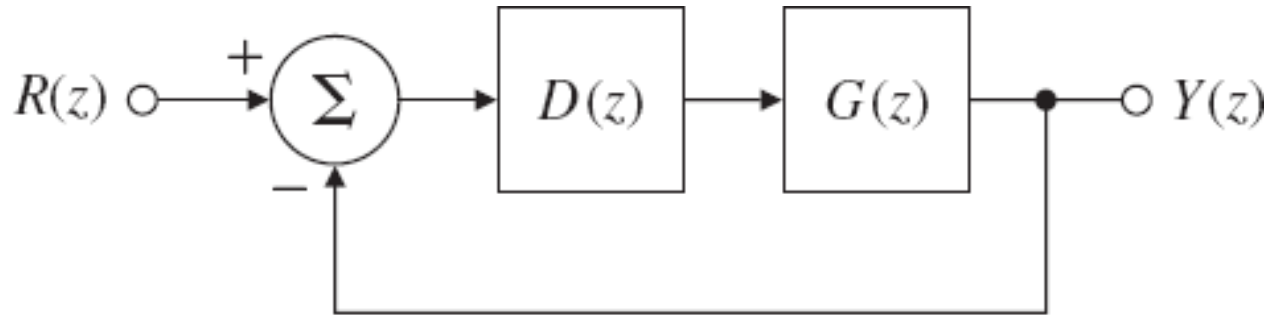
This is analogous to the Laplace transform as a derivative operator: $\mathcal{L}\{\dot{f}(t)\} = sF(s)$

Zero-Order Hold

- A simple model of a typical digital to analog converter:



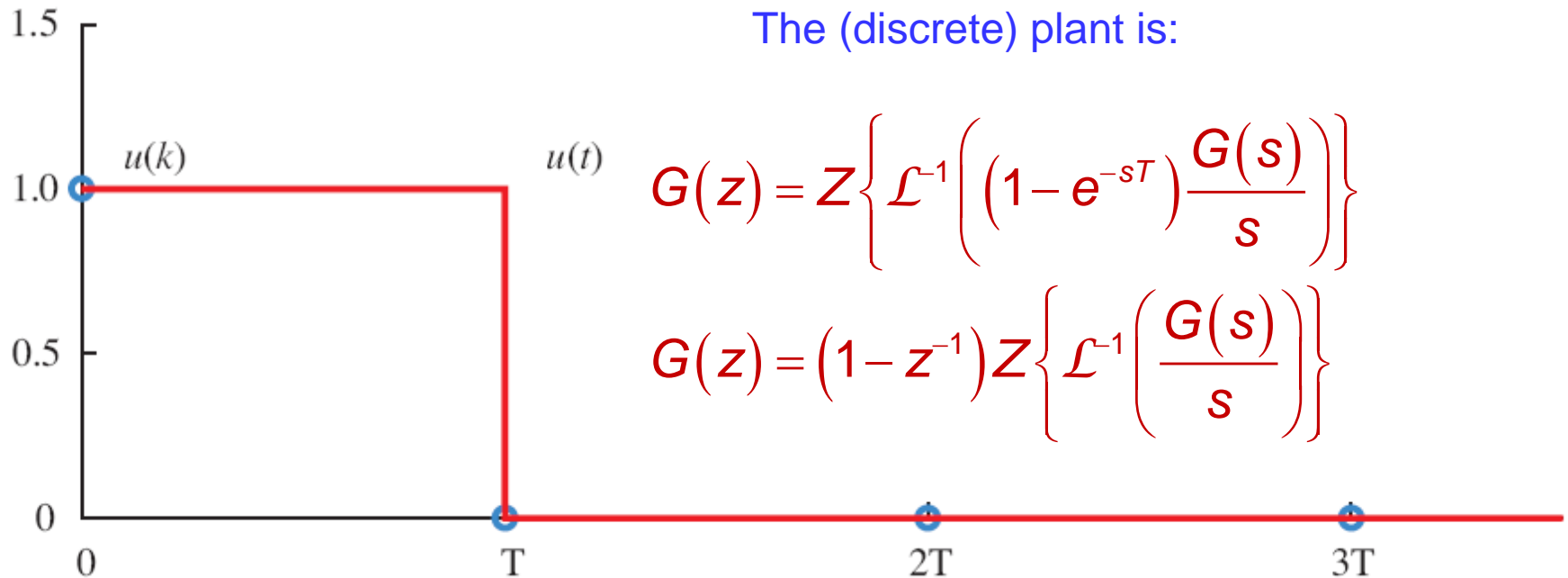
Zero-Order Hold



The (discrete) plant is:

$$G(z) = Z \left\{ \mathcal{L}^{-1} \left((1 - e^{-sT}) \frac{G(s)}{s} \right) \right\}$$

$$G(z) = (1 - z^{-1}) Z \left\{ \mathcal{L}^{-1} \left(\frac{G(s)}{s} \right) \right\}$$



Relation between z and s

- Consider the continuous signal:

$$f(t) = e^{-at}, t > 0$$

- The corresponding Laplace transform is:

$$F(s) = \frac{1}{s+a}$$

- The corresponding z-transform is:

$$F(z) = \frac{z}{z - e^{-aT}}$$

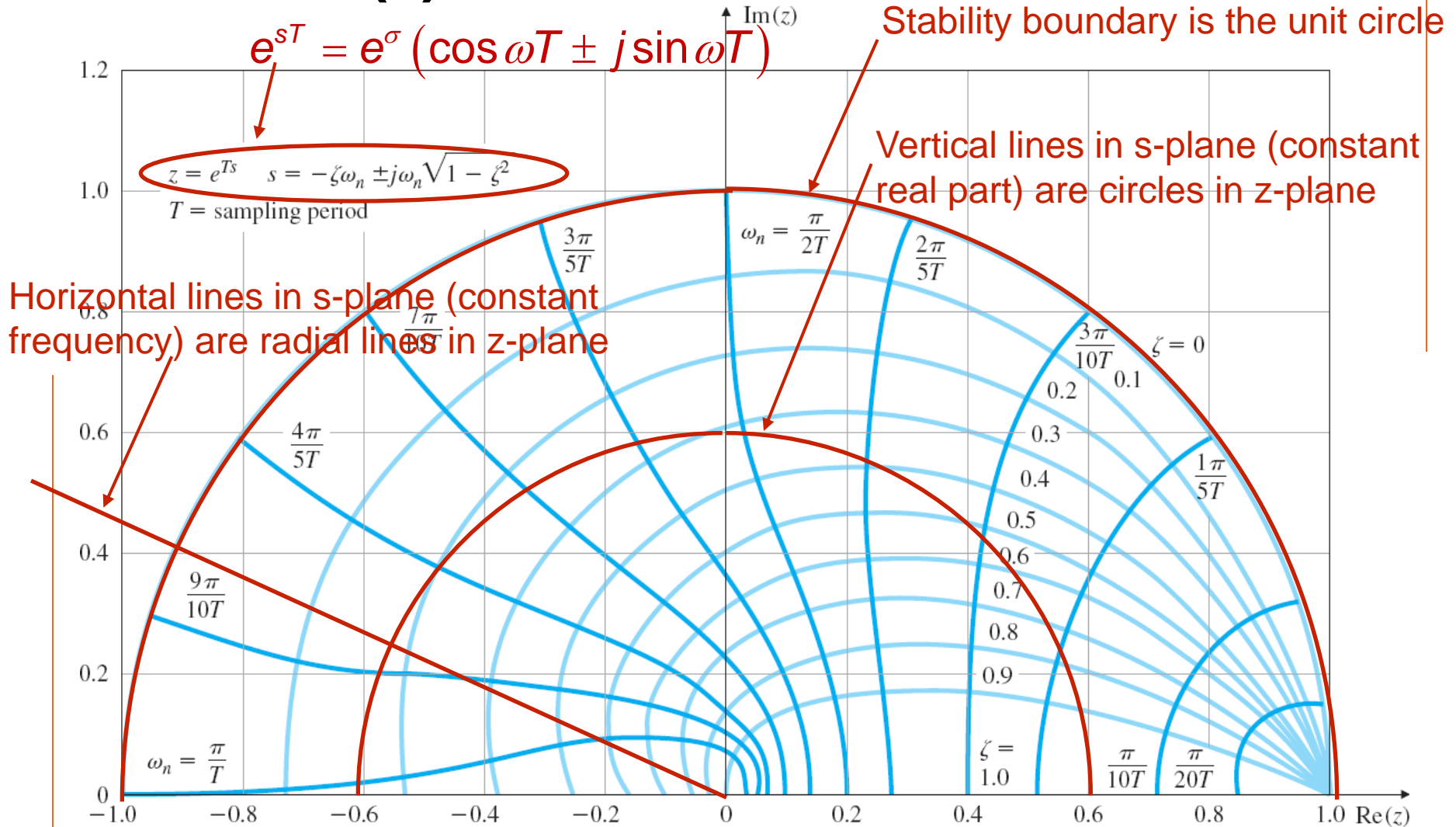
$$\begin{aligned} Z[f(kt)] &= \sum_{k=0}^{\infty} e^{-akT} z^{-k} \\ &= 1 + \left(\frac{e^{-aT}}{z}\right)^1 + \left(\frac{e^{-aT}}{z}\right)^2 + \dots \\ &= \frac{1}{1 - \left(\frac{e^{-aT}}{z}\right)} \end{aligned}$$

A pole at $s = -a$ in the s-plane corresponds to a pole at $z = e^{-aT}$ in the z-plane

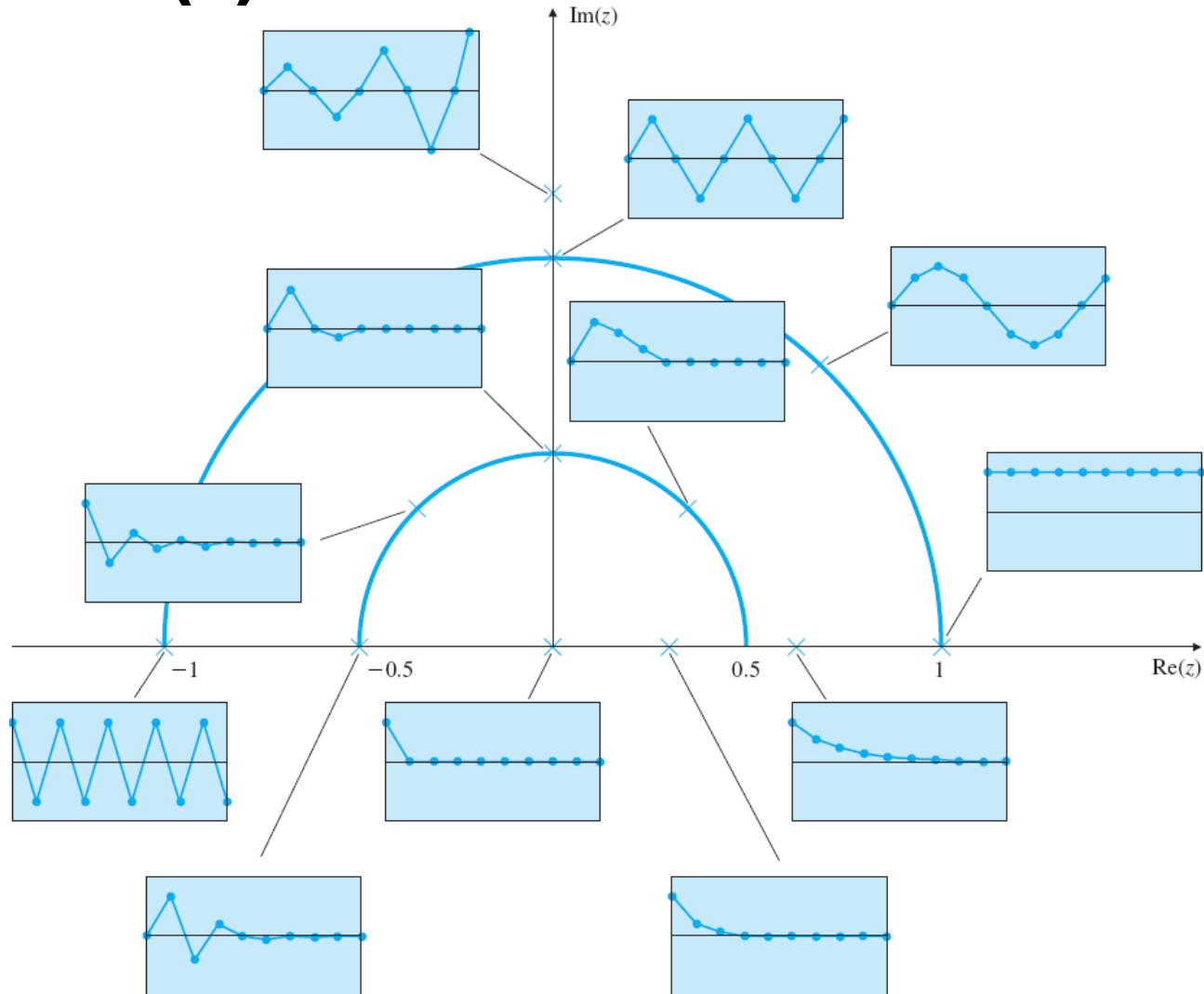
- In general, characteristics in the z-plane are related to those in the s-plane by:

$$z = e^{sT}$$

Effect of (z) Pole Positions



Effect of (z) Pole Positions



Discrete Analysis

