

Find your way in life: listen to Crickets

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1 Intro

The Cricket system is formed by a collection of transmitters, called beacons, and a receiver used to determine your position. Several times a second, a beacon emits a signal; this signal has two parts, each sent at the same time. The first part is a radio message that contains the name of the beacon; since this travels at the speed of light, it is nearly instantaneous, and the receiver can use it to determine when the signal was sent. The second part of the signal is an ultrasonic chirp; this signal travels at the speed of sound, and the receiver measures its travel time to determine the distance between itself and the transmitting beacon.

2 Equations

Each measurement from the cricket system gives you another hint about your position. Specifically, it tells you the distance between your receiver and the transmitting beacon. Since the beacons are mounted in fixed positions, we can use this distance to solve for our location.

Let i index the sequence of Cricket readings, d_i be the distance measurement, (x_i, y_i, z_i) be the beacon's position, and (x, y, z) be the receiver's position. Then by the Pythagorean theorem, each measurement gives us an equation about our position, as shown in Equation 1.

$$\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} = d_i \quad (1)$$

The square root in this equation is decidedly nonlinear in nature; that makes the equations much harder to solve for. By squaring both sides, we

get Equation 2.

$$(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 = d_i^2 \quad (2)$$

Now multiply out and collect terms by their relative power:

$$x_i^2 + y_i^2 + z_i^2 - 2x_ix - 2y_iy - 2z_iz + x^2 + y^2 + z^2 = d_i^2 \quad (3)$$

Looking at Equation 3, there are three types of terms based on how they affect the receiver's position. Terms which are linear in (x, y, z) will be useful when solving for the receiver's position. Terms which only contain (x, y, z) do not change with different Cricket readings; thus they may be collected into a constant, $c = x^2 + y^2 + z^2$. The remaining terms only contain information which was obtained from the distance measurement; they may be moved to the right-hand side of the equation. This sorting results in Equation 4.

$$-2x_ix - 2y_iy - 2z_iz + c = d_i^2 - x_i^2 - y_i^2 - z_i^2 \quad (4)$$

Multiply this by $-1/2$ to obtain the final formula:

$$x_ix + y_iy + z_iz + c = \frac{1}{2}(x_i^2 + y_i^2 + z_i^2 - d_i^2) \quad (5)$$

3 Solving the equations

Each distance reading yields an equation of the form shown above in Equation 5. While these equations could be solved with as few as four readings, such a solution would be very sensitive to errors in the measured distances. To reduce this sensitivity, more than four readings are used when solving the equations. With more equations than unknowns, some error will occur in our solution of each equation. If the normal equation is $f(x) = 0$, then this residual error will be equivalent to solving $f(x) = \epsilon$, where ϵ is some small number.

The least-squares solution to this problem assigns an ϵ_i to each of the equations in the system. It then finds the (x, y, z) values which minimize $\sum_i \epsilon_i^2$, hence the term "least-squares".

To obtain a least-squares solution to the equations, they are first written in matrix form $AX = B$, where A holds the linear coefficients of (x, y, z) and

the constant c , X is a 4-entry vector that will hold the solution (x, y, z, c) , and B holds the right-hand side of Equation 5. Visually, this looks like

$$A = \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ \vdots & & & \\ x_n & y_n & z_n & 1 \end{bmatrix} \quad (6)$$

$$X = \begin{bmatrix} x \\ y \\ z \\ c \end{bmatrix} \quad (7)$$

$$B = \begin{bmatrix} (x_1^2 + y_1^2 + z_1^2 - d_1^2)/2 \\ (x_2^2 + y_2^2 + z_2^2 - d_2^2)/2 \\ \vdots \\ (x_n^2 + y_n^2 + z_n^2 - d_n^2)/2 \end{bmatrix} \quad (8)$$

There are a few standard methods for obtaining a least-squares solution. The two most common use QR or SVD decompositions of the A matrix into an easily solved form. QR factorization is the faster method, but it tends to amplify errors when the A matrix is ill-conditioned. The SVD is slower but more robust to error. Since we have a small system to solve and the DSP is fast enough, we opted to use the SVD for solving the cricket equations.

4 Velocity estimation

The previous sections assume that the receiver is stationary when it receives measurements from the cricket system. When placed on a mobile robot, that assumption is generally false. In order to correct that, a different model must be fit to the data.

Instead of fitting a stationary position, now consider a current position and velocity. Knowing both (x, y, z) and $(\dot{x}, \dot{y}, \dot{z})$ gives us enough information to predict past or future positions: $(x(t), y(t), z(t)) = (x, y, z) + (\dot{x}, \dot{y}, \dot{z})\Delta t$, where Δt is the difference in time from the present moment to when the signal was received.

Using this new system model follows exactly the same steps as before: square the Pythagorean formula, multiply out, and collect terms. This time,

three new constants appear with varying powers of Δt . This is outlined below without further explanation. It may be beneficial to work through this by hand to see how things work.

$$\sqrt{(x_i - (x + \dot{x}\Delta t_i))^2 + (y_i - (y + \dot{y}\Delta t_i))^2 + (z_i - (z - \dot{z}\Delta t_i))^2} = d_i \quad (9)$$

$$x_i x + y_i y + z_i z - x_i \Delta t_i \dot{x} - y_i \Delta t_i \dot{y} - z_i \Delta t_i \dot{z} + c_1 + c_2 \Delta t_i + c_3 \Delta t_i^2 = \frac{1}{2}(x_i^2 + y_i^2 + z_i^2 - d_i^2) \quad (10)$$

$$A = \begin{bmatrix} x_1 & y_1 & z_1 & -x_1 \Delta t_1 & -y_1 \Delta t_1 & -z_1 \Delta t_1 & 1 & \Delta t_1 & \Delta t_1^2 \\ x_2 & y_2 & z_2 & -x_2 \Delta t_2 & -y_2 \Delta t_2 & -z_2 \Delta t_2 & 1 & \Delta t_2 & \Delta t_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & z_n & -x_n \Delta t_n & -y_n \Delta t_n & -z_n \Delta t_n & 1 & \Delta t_n & \Delta t_n^2 \end{bmatrix} \quad (11)$$

$$X = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} (x_1^2 + y_1^2 + z_1^2 - d_1^2)/2 \\ (x_2^2 + y_2^2 + z_2^2 - d_2^2)/2 \\ \vdots \\ (x_n^2 + y_n^2 + z_n^2 - d_n^2)/2 \end{bmatrix} \quad (13)$$

As you can see, the least-squares equations increased from four variables to nine; this model therefore requires more data to obtain the same quality fit. However, when the receiver is moving in a straight line, this model will achieve better tracking than the stationary model.

5 A filtering approach

In the previous sections, all the data points were solved together to obtain a least-squares estimate of the receiver position. Another approach is to use one filter per beacon; each filter will simply estimate the distance from the

receiver to a single beacon as a function of time. A least-squares fit will then take the current position from each of these filters to obtain the receiver's position. In doing so, it will weight each beacon according to the filter's estimate of the beacon's accuracy.

6 Calibration

6.1 Beacon placement

6.2 Establishing a coordinate frame