Feedforward Control: See section 6.4 of Spong Robot Modeling & Control

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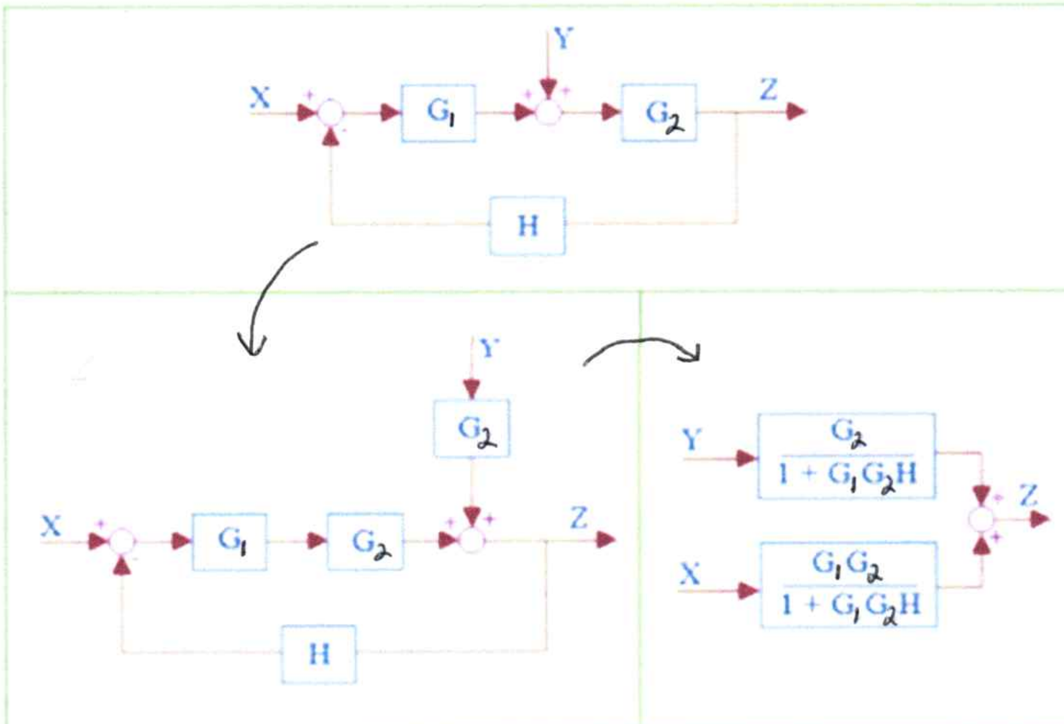

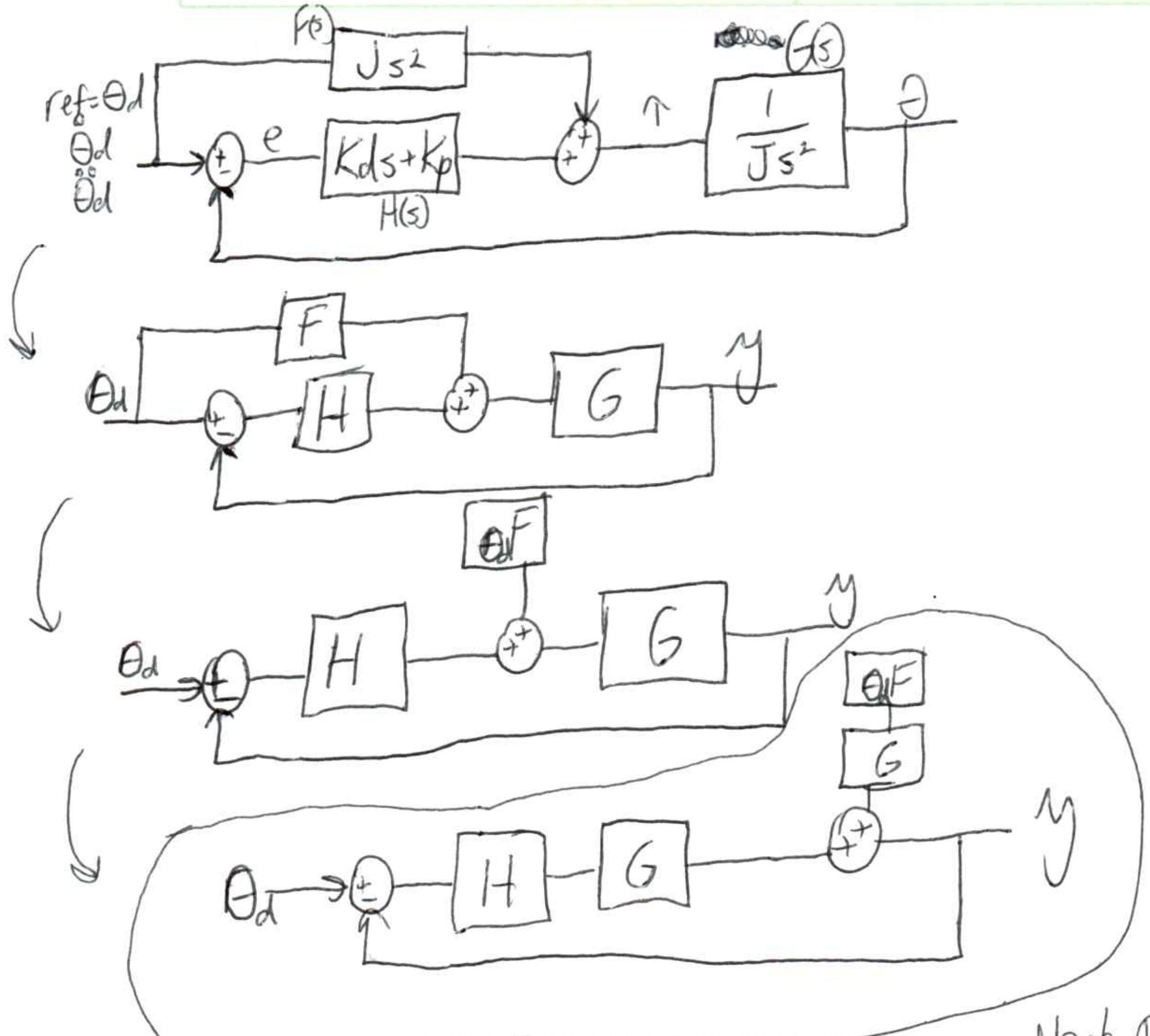

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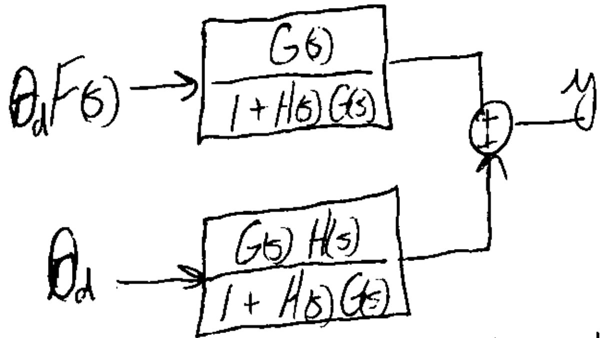

$$= \frac{F(s)G(s)}{1+H(s)G(s)}\theta_d + \frac{H(s)G(s)}{1+H(s)G(s)}\theta_d = y$$

$$y = \left(\frac{F(s)G(s) + H(s)G(s)}{1+H(s)G(s)}\right)\theta_d$$

$$G = \frac{q(s)}{p(s)} \quad H = \frac{c(s)}{d(s)} \quad F = \frac{a(s)}{b(s)}$$

$$y = \frac{\left(\frac{a}{b}\frac{q}{p} + \frac{c}{d}\frac{q}{p}\right)}{\left(1 + \frac{c}{d}\frac{q}{p}\right)}\theta_d$$

$$\frac{pd\ \frac{aq}{bp} + \frac{cq}{dp}}{pd\ 1 + \frac{qc}{pd}}$$

$$\frac{\frac{daq}{b} + cq}{pd + qc}$$

$$\frac{daq + bqc}{bpd + bqc} = \frac{Y(s)}{\theta_d(s)}$$

$$G(s) = \frac{q(s)}{p(s)} = \frac{1}{Js^2} \quad F(s) = \frac{Js^2}{1} = \frac{a(s)}{b(s)}$$

$$H(s) = \frac{c(s)}{d(s)} = \frac{K_d s + K_p}{1}$$

$$\frac{1\cdot(Js^2)\cdot 1 + 1\cdot 1\cdot(K_d s + K_p)}{1\cdot(Js^2)\cdot 1 + 1\cdot 1\cdot(K_d s + K_p)} = \frac{Y(s)}{\theta_d(s)}$$

$$\frac{\theta_d(s)(Js^2 + K_d s + K_p) = Y(s)(Js^2 + K_d s + K_p)}{\theta_d(s) = Y(s)}$$

$$E(s) = \theta_d(s) - Y(s)$$

$$\boxed{(Js^2 + K_d s + K_p)E(s) = 0}$$

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(3) So with Feedforward plus feedback control we have that $\Theta_d(s) = Y(s)$ & $(Js^2 + K_d s + K_p)E(s) = 0$

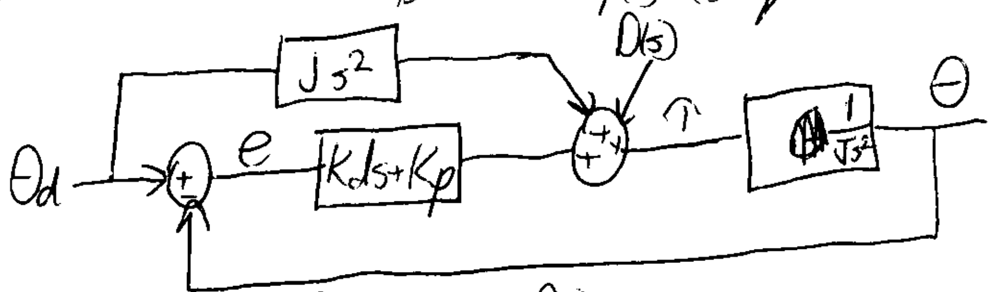
Then - If the model of the robot dynamics is exact and there are no initial state errors - the robot follows the desired trajectory exactly.

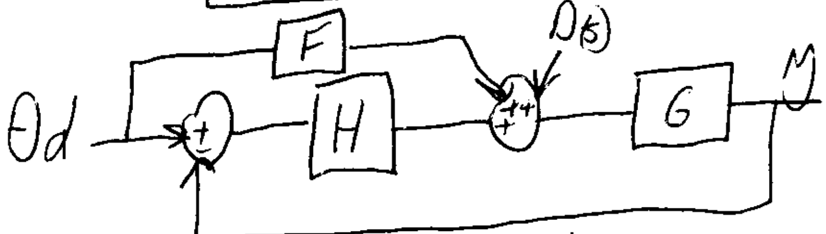
But we know there are errors ~~and~~ and disturbances but $(Js^2+K_ds+K_p)E(s) = 0$ shows that the robot will converge to 0 ← error at the rate of the poles of $Js^2+K_ds+K_p$

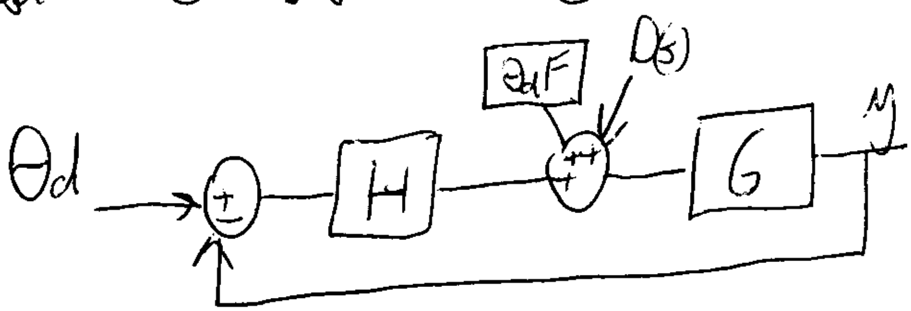
This is better shown by looking at the transfer function given a disturbance

Again using block diagram manipulations we can show that given a disturbance $d(t) \to D(s)$

(Equation 6.35) $\underset{\text{Error}}{E(s)} = \dfrac{-q(s)d(s)}{p(s)d(s)+q(s)c(s)} D(s) \to \dfrac{-1}{Js^2+K_ds+K_p} D(s)$







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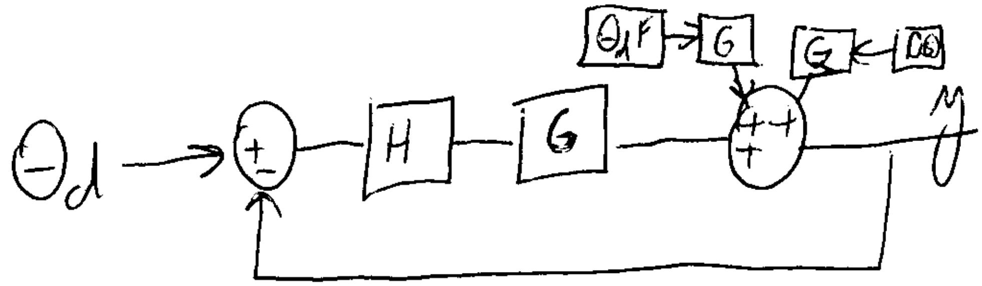

$$\frac{G}{1+HG}D(s) + \frac{FG}{1+HG}\Theta_d(s) + \frac{HG}{1+HG}\Theta_d(s) = Y(s)$$

$$\frac{G}{1+HG}D(s) + \frac{FG+HG}{1+HG}\Theta_d(s) = Y(s)$$

Now we want to find $E(s) = \Theta_d(s) - Y(s)$ in this equation

$$\frac{FG+HG}{1+HG}\Theta_d(s) - Y(s) = \frac{-G}{1+HG}D(s)$$

we need $\frac{FG+HG}{1+HG}$ here so we have error

we can write

$$\frac{FG+HG}{1+HG}\Theta_d(s) - \left(\frac{1+HG}{1+HG}\right)Y(s) = \frac{-G}{1+HG}D(s)$$

$$\frac{FG+HG}{1+HG}Y(s) = \frac{1+HG}{1+HG}Y(s) + \frac{FG-1}{1+HG}Y(s)$$

so $$\frac{1+HG}{1+HG}Y(s) = \frac{FG+HG}{1+HG}Y(s) - \frac{FG-1}{1+HG}Y(s)$$

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substitute this in for $\frac{1+HG}{1+HG}$

(5) so

$$\frac{FG+HG}{1+HG}\Theta_d(s) - \frac{FG+HG}{1+HG}Y(s) + \frac{FG-1}{1+HG}Y(s) = \frac{-G}{1+HG}D(s)$$

$$\frac{FG+HG}{1+HG}E(s) + \underbrace{\frac{FG-1}{1+HG}Y(s)}_{0} = \frac{-G}{1+HG}D(s)$$

$G = \frac{q(s)}{p(s)}$ $H = \frac{c(s)}{d(s)}$ $F = \frac{a(s)}{b(s)} = \frac{p(s)}{q(s)}$ $\Downarrow \frac{1}{G(s)}$

and then how we choose feedforward

$a(s) = p(s)$ $b(s) = q(s)$

$$\frac{\frac{\cancel{p}}{\cancel{q}}\frac{\cancel{q}}{\cancel{p}}^{1} + \frac{cq}{dp}}{1+\frac{c}{d}\frac{q}{p}} + \frac{\cancel{\frac{p}{q}\frac{q}{p}-1}^{\,0}}{1+\frac{c}{d}\frac{q}{p}} = \frac{-\,q/p}{1+\frac{c}{d}\,q/p}$$

$$\frac{1\,\cancel{dp+cq}}{\cancel{dp+cq}}E(s) = \frac{-qd}{dp+cq}D(s)$$

$$E(s) = \frac{-qd}{dp+cq}D(s)$$

$$\text{or } E(s) = \frac{-1}{Js^2+K_ds+K_p}D(s)$$

so error converges to 0 at the rabe of the poles of $Js^2+K_ds+K_p$ given a disturbance $d(t)$